

How accurately can 21 cm tomography constrain cosmology?

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There is growing interest in using 3-dimensional neutral hydrogen mapping with the redshifted 21 cm line as a cosmological probe. However, its utility depends on many assumptions. To aid experimental planning and design, we quantify how the precision with which cosmological parameters can be measured depends on a broad range of assumptions, focusing on the 21 cm signal from $6 < z < 20$. We cover assumptions related to modeling of the ionization power spectrum, to the experimental specifications like array layout and detector noise, to uncertainties in the reionization history, and to the level of contamination from astrophysical foregrounds. We derive simple analytic estimates for how various assumptions affect an experiment's sensitivity, and we find that the modeling of reionization is the most important, followed by the array layout. We present an accurate yet robust method for measuring cosmological parameters that exploits the fact that the ionization power spectra are rather smooth functions that can be accurately fit by 7 phenomenological parameters. We find that for future experiments, marginalizing over these nuisance parameters may provide almost as tight constraints on the cosmology as if 21 cm tomography measured the matter power spectrum directly. A future square kilometer array optimized for 21 cm tomography could improve the sensitivity to spatial curvature and neutrino masses by up to two orders of magnitude, to $\Delta\Omega_k \approx 0.0002$ and $\Delta m_\nu \approx 0.007$ eV, and give a 4σ detection of the spectral index running predicted by the simplest inflation models.

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I. INTRODUCTION

Three-dimensional mapping of our Universe using the redshifted 21 cm hydrogen line has recently emerged as a promising cosmological probe, with arguably greater long-term potential than the cosmic microwave background (CMB). The information garnered about cosmological parameters grows with the volume mapped, so the ultimate goal for the cosmology community is to map our entire horizon volume, the region from which light has had time to reach us during the 14 billion years since our Big Bang. Figure 1 illustrates that whereas the CMB mainly probes a thin shell from $z \sim 1000$, and current large-scale structure probes (like galaxy clustering, gravitational lensing, type Ia supernovae and the Lyman α forest) only map small volume fractions nearby, neutral hydrogen tomography is able to map most of our horizon volume.

Several recent studies have forecast the precision with which such 21 cm tomography can constrain cosmological parameters, both by mapping diffuse hydrogen before and during the reionization epoch [1, 2, 3] and by mapping neutral hydrogen in galactic halos after reion-

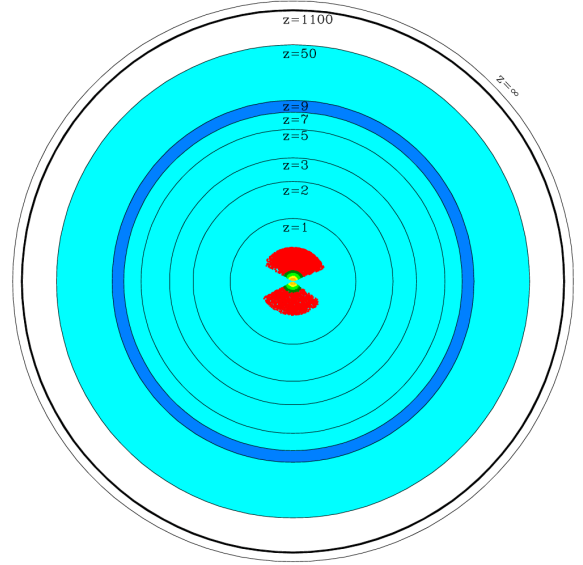


FIG. 1: 21 cm tomography can potentially map most of our observable universe (light blue/light grey), whereas the CMB probes mainly a thin shell at $z \sim 10^3$ and current large-scale structure surveys (here exemplified by the Sloan Digital Sky Survey and its luminous red galaxies) map only small volumes near the center. This paper focuses on the convenient $7 \lesssim z \lesssim 9$ region (dark blue/dark grey).

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ization [4]. These studies find that constraints based on

TABLE I: Factors that affect the cosmological parameter measurement accuracy.

| Assumptions | | Pessimistic | Middle | Optimistic |
|----------------|--|--|--|--|
| Power modeling | Ionization power spectrum modeling | Marginalize over arbitrary $P_{\mu 0}$ and $P_{\mu 2}$ | Marginalize over constants that parametrize $\mathcal{P}_{xx}(k)$ and $\mathcal{P}_{x\delta}(k)$ | No ionization power spectrum, $\mathcal{P}_{\delta\delta}(k) \propto P_{\Delta T}(\mathbf{k})$. |
| | Non-linear cut-off scale k_{\max} | 1 Mpc ⁻¹ | 2 Mpc ⁻¹ | 4 Mpc ⁻¹ |
| | Non-Gaussianity of ionization signals | Doubles sample variance | Negligible | |
| Cosmological | Reionization history | Gradual reionization over wide range of redshifts | | Abrupt reionization at $z \lesssim 7$ |
| | Redshift range | 7.3-8.2 | 6.8 – 8.2 | 6.8 - 10 |
| | Parameter space | Vanilla model plus optional parameters | | Vanilla model parameters |
| Experimental | Data | MWA, LOFAR, 21CMA | Intermediate case | SKA, FFTT |
| | Array configuration ^a | $\eta = 0.15$ | $\eta = 0.8, n = 2$ | Giant core |
| | Collecting area ^b | $0.5 \times$ design values | Design values | $2 \times$ Design values |
| | Observation time ^c | 1000 hours | 4000 hours | 16000 hours |
| | System temperature | $2 \times T_{\text{sys}}$ in [29] | T_{sys} given in [29] | $0.5 \times T_{\text{sys}}$ in [29] |
| Astrophysical | Residual foregrounds cut-off scale k_{\min} ^d | $4\pi/yB$ | $2\pi/yB$ | π/yB |

^aFor the FFTT, we consider only the case where all dipoles are in a giant core.

^bSee designed or assumed values of A_e in Table IV.

^cAssumes observation of two places in the sky.

^dIt is hard to predict the level of the residual foregrounds after the removal procedure. To quantify contributions from other factors, we take the approximation that there is no residual foregrounds at $k > k_{\min}$. Here in the table, yB is the comoving (l.o.s.) distance width of a single z -bin.

TABLE II: The dependence of cosmological constraints on the full range of assumptions. We assume the fiducial values given in Section IIF, and employ the Fisher matrix formalism to forecast the 1σ accuracy of 21cm tomography measurements. Unless otherwise noted, errors are computed by marginalizing over all other parameters in the first ten columns (which we refer to as the “vanilla” parameters). In “All OPT/MID/PESS”, we use the assumptions of the right, middle and left column of Table I, respectively. We assume that the total observing time is split between two sky regions, each for an amount in Table I, using a giant/quasi-giant/small core array configuration where 100%/80%/15% of the antennae in the inner core are compactly laid at the array center while the rest 0%/20%/85% of antennae fall off in density as $\rho \sim r^{-2}$ outside the compact core.

| | | Vanilla Alone | | | | | | | | | | | | |
|--------------------|----------|------------------|----------------------------|----------------------------|--------------|------------------|---------------|--------------------------------------|-------------------------|-------------------------|-------------------------|------------------|---------------------|----------------|
| | | $\Delta\Omega_b$ | $\Delta \ln(\Omega_m h^2)$ | $\Delta \ln(\Omega_b h^2)$ | Δn_s | $\Delta \ln A_s$ | $\Delta \tau$ | $\Delta \bar{x}_H(7.0)$ ^a | $\Delta \bar{x}_H(7.5)$ | $\Delta \bar{x}_H(8.0)$ | $\Delta \bar{x}_H(9.2)$ | $\Delta\Omega_k$ | Δm_ν [eV] | $\Delta\alpha$ |
| Planck | | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | ... | ... | ... | 0.025 | 0.23 | 0.0026 |
| +LOFAR | All OPT | 0.0044 | 0.0052 | 0.0051 | 0.0018 | 0.0087 | 0.0042 | 0.0063 | 0.0063 | 0.0063 | 0.0063 | 0.0022 | 0.023 | 0.00073 |
| | All MID | 0.0070 | 0.0081 | 0.0059 | 0.0032 | 0.0088 | 0.0043 | 0.18 | 0.26 | 0.23 | ... | 0.018 | 0.22 | 0.0026 |
| | All PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | 51 | 49 | ... | 0.025 | 0.23 | 0.0026 |
| +MWA | All OPT | 0.0063 | 0.0074 | 0.0055 | 0.0024 | 0.0087 | 0.0043 | 0.0062 | 0.0062 | 0.0062 | 0.0062 | 0.0056 | 0.017 | 0.00054 |
| | All MID | 0.0061 | 0.0070 | 0.0056 | 0.0030 | 0.0087 | 0.0043 | 0.32 | 0.22 | 0.29 | ... | 0.021 | 0.19 | 0.0026 |
| | All PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | 29 | 30 | ... | 0.025 | 0.23 | 0.0026 |
| +SKA | All OPT | 0.00052 | 0.0018 | 0.0040 | 0.00039 | 0.0087 | 0.0042 | 0.0059 | 0.0059 | 0.0059 | 0.0059 | 0.0011 | 0.010 | 0.00027 |
| | All MID | 0.0036 | 0.0040 | 0.0044 | 0.0025 | 0.0087 | 0.0043 | 0.0094 | 0.014 | 0.011 | ... | 0.0039 | 0.056 | 0.0022 |
| | All PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | 1.1 | 1.0 | ... | 0.025 | 0.23 | 0.0026 |
| +FFTT ^b | All OPT | 0.00010 | 0.0010 | 0.0029 | 0.000088 | 0.0086 | 0.0042 | 0.0051 | 0.0051 | 0.0051 | 0.0051 | 0.00020 | 0.0018 | 0.000054 |
| | All MID | 0.00038 | 0.00034 | 0.00059 | 0.00033 | 0.0086 | 0.0042 | 0.0013 | 0.0022 | 0.0031 | ... | 0.00023 | 0.0066 | 0.00017 |
| | All PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | 0.0043 | 0.0047 | ... | 0.025 | 0.11 | 0.0024 |

^a $\bar{x}_H(z)$ refers to the mean neutral fraction at redshift z .

^bFFTT stands for Fast Fourier Transform Telescope, a future square kilometer array optimized for 21 cm tomography as described in [11]. Dipoles in FFTT are all in a giant core, and this configuration does not vary.

the cosmic microwave background measurements can be significantly improved. However, all of these papers make various assumptions, and it is important to quantify to what extent their forecasts depend on these assumptions. This issue is timely because 21 cm experiments (like LOFAR [7], 21CMA [8], MWA [9] and SKA [10]) are still largely in their planning, design or construction phases. These experiments will be described in detail in Section II G. In order to maximize their scientific “bang for the buck”, it is therefore important to quantify how various design tradeoffs affect their sensitivity to cosmological parameters.

The reason that neutral hydrogen allows mapping in three rather than two dimensions is that the redshift of the 21 cm line provides the radial coordinate along the line-of-sight (l.o.s.). This signal can be observed from the so-called dark ages [15, 16] before any stars had formed, through the epoch of reionization (EoR), and even to the current epoch (where most of the neutral hydrogen is confined within galaxies). We focus in this study on the 21 cm signal from $6 < z < 20$ – the end of the dark ages through the EoR. This is the redshift range at which the synchrotron foregrounds are smallest, and consequently is the range most assessable for all planned 21 cm arrays.

There are three position-dependent quantities that imprint signatures on the 21 cm signal: the hydrogen density, the neutral fraction, and the spin temperature. For cosmological parameter measurements, only the first quantity is of interest, and the last two are nuisances. (For some astronomical questions, the situation is reversed.) The 21 cm spin-flip transition of neutral hydrogen can be observed in the form of either an absorption line or an emission line against the CMB blackbody spectrum, depending on whether the spin temperature is lower or higher than the CMB temperature.

During the epoch of reionization, the spin temperature is likely coupled to the gas temperature through Ly α photons via the Wouthuysen-Field Effect [12, 13], and the gas in the inter-galactic medium (IGM) has been heated by X-ray photons to hundreds of Kelvin from the first stars [14]. If this is true, the 21cm signal will only depend on the hydrogen density and the neutral fraction. However, astrophysical uncertainties prevent a precise prediction for exactly when the gas is heated to well above the CMB temperature and is coupled to the spin temperature. In this paper, we follow [1, 2] and focus entirely on the regime when the spin temperature is much larger than the CMB temperature [17, 18, 19], such that the observed signal depends only on fluctuations in density and/or the neutral fraction. Specifically, we focus on the time interval from when this approximation becomes valid (around the beginning of the reionization [17, 18, 19]) until most hydrogen has become ionized, illustrated by the darkest region in Figure 1. Despite this simplification, the methods that we apply to model the ionization fluctuations almost certainly can be applied to model spin temperature fluctuations with minimal additional free parameters.

In Table I, we list all the assumptions that affect the

accuracy of cosmological parameter measurements, including ones about power modeling, cosmology, experimental design, and astrophysical foregrounds. For each case, we provide three categories of assumptions: one pessimistic (PESS), one middle-of-the-road (MID) and one optimistic (OPT). Since we wish to span the entire range of uncertainties, we have made both the PESS and OPT models rather extreme. The MID model is intended to be fairly realistic, but somewhat on the conservative (pessimistic) side.

Before describing these assumptions in detail in the next section, it is important to note that taken together, they make a huge difference. Table II illustrates this by showing the cosmological parameter constraints resulting from using all the OPT assumptions, all the MID assumptions or all the PESS assumptions, respectively. For example, combining CMB data from Planck and 21 cm data from FFTT, the 1σ uncertainty differs by a factor of 125 for Ω_k and by a factor of 61 for m_ν depending on assumptions. It is therefore important to sort out which of the assumptions contribute the most to these big discrepancies, and which assumptions do not matter much. This is a key goal of our paper.

The rest of this paper is organized as follows. In Section II, we explain in detail the assumptions in the same order as in Table I, and also present a new method for modeling the ionization power spectra. In Section III, we quantify how the cosmological parameter measurement accuracy depends on each assumption, and we derive simple analytic approximations of these relations. In Section IV, we conclude with a discussion of the relative importance of these assumptions, and implications for experimental design.

II. FORECASTING METHODS & ASSUMPTIONS

A. Fundamentals of 21cm cosmology

1. Power spectrum of 21 cm radiation

We review the basics of the 21 cm radiation temperature and power spectrum only briefly here, and refer the interested reader to [20] for a more comprehensive discussion of the relevant physics. The difference between the observed 21 cm brightness temperature at the redshifted frequency ν and the CMB temperature T_{CMB} is [22]

$$T_b(\mathbf{x}) = \frac{3c^3 h A_{10} n_H(\mathbf{x}) [T_S(\mathbf{x}) - T_{\text{CMB}}]}{32\pi k_B \nu_0^2 T_S(\mathbf{x}) (1+z)^2 (dv_{\parallel}/dr)}, \quad (1)$$

where T_S is the spin temperature, n_H is the number density of the neutral hydrogen gas, and $A_{10} \approx 2.85 \times 10^{-15} \text{s}^{-1}$ is the spontaneous decay rate of 21cm transition. The factor dv_{\parallel}/dr is the gradient of the physical velocity along the line of sight (r is the comoving distance), which is $H(z)/(1+z)$ on average (i.e. for no peculiar velocity). Here $H(z)$ is the Hubble parameter at redshift

z . The spatially averaged brightness temperature at redshift z is (in units of mK)

$$\bar{T}_b \approx 23.88 \bar{x}_H \left(\frac{\bar{T}_S - T_{\text{CMB}}}{T_S} \right) \left(\frac{\Omega_b h^2}{0.02} \right) \left(\frac{0.15}{\Omega_m h^2} \frac{1+z}{10} \right)^{1/2}, \quad (2)$$

where \bar{x}_H is the mean neutral fraction, and \bar{T}_S is the averaged spin temperature. If $T_S \gg T_{\text{CMB}}$ in the EoR, the 21cm emission should therefore be observed at the level of milli-Kelvins.

To calculate the fluctuations, we rewrite Eq. (1) in terms of δ (the hydrogen mass density fluctuation), δ_x (the fluctuation in the ionized fraction), and the gradient of the peculiar velocity $\partial v_r / \partial r$ along the line of sight, using the fact that $dv_{\parallel} / dr = H(z) / (1+z) + \partial v_r / \partial r$:

$$T_b(\mathbf{x}) = \tilde{T}_b [1 - \bar{x}_i(1 + \delta_x)] (1 + \delta) \left(1 - \frac{1}{Ha} \frac{\partial v_r}{\partial r} \right) \times \left(\frac{T_S - T_{\text{CMB}}}{T_S} \right). \quad (3)$$

Here $\bar{x}_i \equiv 1 - \bar{x}_H$ is the mean ionized fraction, and we have defined $\tilde{T}_b \equiv \bar{T}_b / \bar{x}_H \times [\bar{T}_S / (\bar{T}_S - T_{\text{CMB}})]$. We write $\delta_v \equiv (Ha)^{-1} \partial v_r / \partial r$. In Fourier space, it is straightforward to show that, as long as $\delta \ll 1$ so that linear perturbation theory is valid, $\delta_v(\mathbf{k}) = -\mu^2 \delta$, where $\mu = \hat{\mathbf{k}} \cdot \hat{\mathbf{n}}$ is the cosine of the angle between the Fourier vector \mathbf{k} and the line of sight. In this paper, we restrict our attention to the linear regime. We will also throughout this paper assume $T_S \gg T_{\text{CMB}}$ during the EoR, making the last factor in Eq. (3) unity for the reasons detailed in Section I.

In Fourier space, the power spectrum $P_{\Delta T}(\mathbf{k})$ of the 21cm fluctuations is defined by $\langle \Delta T_b^*(\mathbf{k}) \Delta T_b(\mathbf{k}') \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k} - \mathbf{k}') P_{\Delta T}(\mathbf{k})$, where ΔT_b is the deviation from the mean brightness temperature. It is straightforward to show from Eq. (3) that, to leading order,

$$P_{\Delta T}(\mathbf{k}) = \tilde{T}_b^2 \{ [\bar{x}_H^2 P_{\delta\delta} - 2\bar{x}_H P_{x\delta} + P_{xx}] + 2\mu^2 [\bar{x}_H^2 P_{\delta\delta} - \bar{x}_H P_{x\delta}] + \mu^4 \bar{x}_H^2 P_{\delta\delta} \}. \quad (4)$$

Here $P_{xx} = \bar{x}_i^2 P_{\delta_x \delta_x}$ and $P_{x\delta} = \bar{x}_i P_{\delta_x \delta}$ are the ionization power spectrum and the density-ionization power spectrum respectively. For convenience, we define $\mathcal{P}_{\delta\delta}(k) \equiv \tilde{T}_b^2 \bar{x}_H^2 P_{\delta\delta}(k)$, $\mathcal{P}_{x\delta}(k) \equiv \tilde{T}_b^2 \bar{x}_H P_{x\delta}(k)$ and $\mathcal{P}_{xx}(k) \equiv \tilde{T}_b^2 P_{xx}(k)$, so the total 21 cm power spectrum can be written as three terms with different angular dependence:

$$P_{\Delta T}(\mathbf{k}) = P_{\mu^0}(k) + P_{\mu^2}(k) \mu^2 + P_{\mu^4}(k) \mu^4, \quad (5)$$

where

$$P_{\mu^0} = \mathcal{P}_{\delta\delta} - 2\mathcal{P}_{x\delta} + \mathcal{P}_{xx}, \quad (6)$$

$$P_{\mu^2} = 2(\mathcal{P}_{\delta\delta} - \mathcal{P}_{x\delta}), \quad (7)$$

$$P_{\mu^4} = \mathcal{P}_{\delta\delta}. \quad (8)$$

Since P_{μ^4} involves only the matter power spectrum that depends only on cosmology, Barkana and Loeb [23, 24]

argued that in principle, one can separate cosmology from astrophysical “contaminants” such as \mathcal{P}_{xx} and $\mathcal{P}_{x\delta}$ whose physics is hitherto far from known. We will quantify the accuracy of this conservative approach (which corresponds to our PESS scenario for ionization power spectrum modeling below) in Section III.

2. From \mathbf{u} to \mathbf{k}

The power spectrum $P_{\Delta T}(\mathbf{k})$ and the comoving vector \mathbf{k} (the Fourier dual of the comoving position vector \mathbf{r}) are not directly measured by 21cm experiments. An experiment cannot directly determine which position vector \mathbf{r} a signal is coming from, but instead which vector $\boldsymbol{\Theta} \equiv \theta_x \hat{e}_x + \theta_y \hat{e}_y + \Delta f \hat{e}_z$ it is coming from, where (θ_x, θ_y) give the angular location on the sky plane, and Δf is the frequency difference from the central redshift of a z -bin. For simplicity, we assume that the sky volume observed is small enough that we can linearize the relation between $\boldsymbol{\Theta}$ and \mathbf{r} . Specifically, we assume that the sky patch observed is much less than a radian across, so that we can approximate the sky as flat¹, and that separations in frequency near the mean redshift z_* are approximately proportional to separations in comoving distance. In these approximations, if there are no peculiar velocities,

$$\boldsymbol{\Theta}_{\perp} = \frac{\mathbf{r}_{\perp}}{d_A(z_*)}, \quad (9)$$

$$\Delta f = \frac{\Delta r_{\parallel}}{y(z_*)}. \quad (10)$$

Here “ \perp ” denotes the vector component perpendicular to the line of sight, *i.e.*, in the (x, y) -plane, and d_A is the comoving angular diameter distance given by [43]

$$d_A(z) = \frac{c}{H_0} |\Omega_k|^{-1/2} S \left[|\Omega_k|^{1/2} \int_0^z \frac{dz'}{E(z')} \right], \quad (11)$$

where

$$E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}, \quad (12)$$

is the relative cosmic expansion rate and the function $S(x)$ equals $\sin(x)$ if $\Omega_k < 0$, x if $\Omega_k = 0$, and $\sinh x$ if $\Omega_k > 0$. The conversion factor between comoving distances intervals and frequency intervals is

$$y(z) = \frac{\lambda_{21}(1+z)^2}{H_0 E(z)}, \quad (13)$$

¹ The FFTT is designed for all-sky mapping (*i.e.* the field of view is of order 2π). However, since the angular scales from which we get essentially all our cosmological information are much smaller than a radian (with most information being on arcminute scales), the flat-sky approximation is accurate as long as the data is analyzed separately in many small patches and the constraints are subsequently combined.

where $\lambda_{21} \approx 21$ cm is the rest-frame wavelength of the 21 cm line.

We write the Fourier dual of Θ as $\mathbf{u} \equiv u_x \hat{e}_x + u_y \hat{e}_y + u_{\parallel} \hat{e}_z$ (u_{\parallel} has units of time). The relation between \mathbf{u} and \mathbf{k} is therefore

$$\mathbf{u}_{\perp} = d_A \mathbf{k}_{\perp}, \quad (14)$$

$$u_{\parallel} = y k_{\parallel}. \quad (15)$$

In \mathbf{u} -space, the power spectrum $P_{\Delta T}(\mathbf{u})$ of 21cm signals is defined by $\langle \Delta \tilde{T}_b^*(\mathbf{u}) \Delta \tilde{T}_b(\mathbf{u}') \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{u} - \mathbf{u}') P_{\Delta T}(\mathbf{u})$, and is therefore related to $P_{\Delta T}(\mathbf{k})$ by

$$P_{\Delta T}(\mathbf{u}) = \frac{1}{d_A^2 y} P_{\Delta T}(\mathbf{k}). \quad (16)$$

Note that cosmological parameters affect $P_{\Delta T}(\mathbf{u})$ in two ways: they both change $P_{\Delta T}(\mathbf{k})$ and alter the geometric projection from \mathbf{k} -space to \mathbf{u} -space. If d_A and y changed while $P_{\Delta T}(\mathbf{k})$ remained fixed, the observable power spectrum $P_{\Delta T}(\mathbf{u})$ would be dilated in both the \mathbf{u}_{\perp} and u_{\parallel} directions and rescaled in amplitude, while retaining its shape. Since both d_A and y depend on the three parameters $(\Omega_k, \Omega_{\Lambda}, h)$, and the Hubble parameter is in turn given by the parameters in Table II via the identity $h = \sqrt{\Omega_m h^2 / (1 - \Omega_{\Lambda} - \Omega_k)}$, we see that these geometric effects provide information only about our parameters $(\Omega_k, \Omega_{\Lambda}, \Omega_m h^2)$. Baryon acoustic oscillations in the power spectrum provide a powerful “standard ruler”, and the equations above show that if one generalizes to the dark energy to make Ω_{Λ} an arbitrary function of z , then the cosmic expansion history $H(z)$ can be measured separately at each redshift bin, as explored in [4, 5, 6]. 21 cm tomography information on our other cosmological parameters ($n_s, A_s, \Omega_b h^2, m_{\nu}, \alpha$, etc.) thus comes only from their direct effect on $P_{\Delta T}(\mathbf{k})$. Also note that $(\Omega_k, \Omega_{\Lambda})$ affect $P_{\Delta T}(\mathbf{k})$ only by modulating the rate of linear perturbation growth, so they alter only the amplitude and not the shape of $P_{\Delta T}(\mathbf{k})$.

If we were to use Eq. (16) to infer $P_{\Delta T}(\mathbf{k})$ from the measured power spectrum $P_{\Delta T}(\mathbf{u})$ while assuming incorrect cosmological parameter values, then this geometric scaling would cause the inferred $P_{\Delta T}(\mathbf{k})$ to be distorted by the so-called Alcock-Paczynski (AP) effect [26, 27] and not take the simple form of Eqns.(5)-(8). To avoid this complication, we therefore perform our Fisher matrix analysis directly in terms of $P_{\Delta T}(\mathbf{u})$, since this quantity is directly measurable without any cosmological assumptions.

The above transformations between \mathbf{u} -space and \mathbf{r} -space are valid when there are no peculiar velocities. The radial peculiar velocities v_r that are present in the real world induce the familiar redshift space distortions that were discussed in Section II A 1, causing μ^2 and μ^4 power spectrum anisotropies that were described there.

B. Assumptions about \mathcal{P}_{xx} and $\mathcal{P}_{x\delta}$

During the EoR, ionized bubbles (HII regions) in the IGM grow and eventually merge with one another. Consequently, $\mathcal{P}_{xx}(\mathbf{k})$ and $\mathcal{P}_{x\delta}(\mathbf{k})$ contribute significantly to the total 21cm power spectrum. The study of the forms of these two ionization power spectra has made rapid progress recently, particularly through the semi-analytical calculations [17, 19, 21, 50] and radiative transfer simulations [28, 51]. However, these models depend on theoretically presumed parameters whose values cannot currently be calculated from first principles. From the experimental point of view, it is therefore important to develop data analysis methods that depend only on the most generic features of the ionization power spectra. In this paper, we consider three methods — our OPT, MID and PESS models — that model \mathcal{P}_{xx} and $\mathcal{P}_{x\delta}$ as follows:

$$\text{(OPT)} \quad \begin{cases} \mathcal{P}_{xx}(k) = 0 \\ \mathcal{P}_{x\delta}(k) = 0 \end{cases} \quad (17)$$

(MID)

$$\begin{cases} \mathcal{P}_{xx}(k) = b_{xx}^2 [1 + \alpha_{xx}(k R_{xx}) + (k R_{xx})^2]^{-\frac{2\alpha_{xx}}{2}} \mathcal{P}_{\delta\delta}^{(\text{fid})} \\ \mathcal{P}_{x\delta}(k) = b_{x\delta}^2 \exp[-\alpha_{x\delta}(k R_{x\delta}) - (k R_{x\delta})^2] \mathcal{P}_{\delta\delta}^{(\text{fid})} \end{cases} \quad (18)$$

$$\text{(PESS)} \quad \begin{cases} \mathcal{P}_{xx}(k) = \text{arbitrary} \\ \mathcal{P}_{x\delta}(k) = \text{arbitrary} \end{cases} \quad (19)$$

In the next three subsections, we explain these models in turn.

1. OPT model

It is likely that before reionization started (while $\bar{x}_H = 1$ and $\mathcal{P}_{xx} = \mathcal{P}_{x\delta} = 0$), hydrogen gas had already been sufficiently heated that $T_S \gg T_{\text{CMB}}$. In this regime, Eq. (17) holds. This OPT scenario is clearly the simplest model, since the total 21cm power spectrum is simply proportional to $\mathcal{P}_{\delta\delta}$: $P_{\Delta T}(\mathbf{k}) = \mathcal{P}_{\delta\delta}(k)(1 + \mu^2)^2$. To forecast the 1σ error, we use the Fisher matrix formalism [25]. Repeating the derivation in [44], the Fisher matrix for cosmological parameters λ_a ($a = 1, \dots, N_p$) is

$$\mathbf{F}_{ab} = \frac{1}{2} \int \left(\frac{\partial \ln P_{\Delta T}^{\text{tot}}(\mathbf{u})}{\partial \lambda_a} \right) \left(\frac{\partial \ln P_{\Delta T}^{\text{tot}}(\mathbf{u})}{\partial \lambda_b} \right) V_{\Theta} \frac{d^3 u}{(2\pi)^3}, \quad (20)$$

where the integral is taken over the observed part of \mathbf{u} -space, and $P_{\Delta T}^{\text{tot}}(\mathbf{u})$ denotes the combined power spectrum from cosmological signal and all forms of noise. Here $V_{\Theta} = \Omega \times B$ is the volume of the Θ -space where Ω is the solid angle within the field of view (f.o.v.) and B is the

TABLE III: Fiducial values of ionization parameters adopted for Figure 2. R_{xx} and $R_{x\delta}$ are in units of Mpc, while other parameters are unitless.

| z | \bar{x}_H | b_{xx}^2 | R_{xx} | α_{xx} | γ_{xx} | $b_{x\delta}^2$ | $R_{x\delta}$ | $\alpha_{x\delta}$ |
|-----|-------------|------------|----------|---------------|---------------|-----------------|---------------|--------------------|
| 9.2 | 0.9 | 0.208 | 1.24 | -1.63 | 0.38 | 0.45 | 0.56 | -0.4 |
| 8.0 | 0.7 | 2.12 | 1.63 | -0.1 | 1.35 | 1.47 | 0.62 | 0.46 |
| 7.5 | 0.5 | 9.9 | 1.3 | 1.6 | 2.3 | 3.1 | 0.58 | 2. |
| 7.0 | 0.3 | 77. | 3.0 | 4.5 | 2.05 | 8.2 | 0.143 | 28. |

frequency size of a z -bin. The Fisher matrix determines the parameter errors as $\Delta\lambda_a = \sqrt{(\mathbf{F}^{-1})_{aa}}$.

For computational convenience, we subdivide u -space into pixels so small that the power spectrum is roughly constant in each one, obtaining

$$\mathbf{F}_{ab} \approx \sum_{\text{pixels}} \frac{1}{[\delta P_{\Delta T}(\mathbf{u})]^2} \left(\frac{\partial P_{\Delta T}(\mathbf{u})}{\partial \lambda_a} \right) \left(\frac{\partial P_{\Delta T}(\mathbf{u})}{\partial \lambda_b} \right), \quad (21)$$

where the power spectrum measurement error in a pixel at \mathbf{u} is

$$\delta P_{\Delta T}(\mathbf{u}) = \frac{P_{\Delta T}^{\text{tot}}(\mathbf{u})}{N_c^{1/2}} = \frac{P_{\Delta T}(\mathbf{u}) + P_N(u_{\perp})}{N_c^{1/2}}. \quad (22)$$

Here $P_N(u_{\perp})$ is the noise power spectrum and will be discussed in detail in Section II G 3; note that it is independent of u_{\parallel} and depends only on u_{\perp} through the baseline distribution of the antenna array.

$$N_c = 2\pi k^2 \sin \theta \Delta k \Delta \theta \times \text{Vol} / (2\pi)^3 \quad (23)$$

is the number of independent cells in an annulus summing over the azimuthal angle. We have the factor $\sqrt{1/N_c}$ in $\delta P_{\Delta T}$ instead of the normal $\sqrt{2/N_c}$ because we only sum over half the sphere.

2. MID model

After reionization starts, both ionization power spectra \mathcal{P}_{xx} and $\mathcal{P}_{x\delta}$ make significant contribution to the total 21cm power spectrum. We explore two different analysis methods — our MID and PESS models — for separating the cosmological signal from these astrophysical contaminants (i.e. \mathcal{P}_{xx} and $\mathcal{P}_{x\delta}$).

Our MID model assumes that both ionization power spectra $\mathcal{P}_{xx}(k)$ and $\mathcal{P}_{x\delta}(k)$ are smooth functions of k which can be parametrized by a small number of nuisance parameters $\beta_1, \dots, \beta_{n_{\text{ion}}}$ related to reionization physics. Combining these ionization parameters with our cosmological ones λ_a into a larger parameter set p_{α} ($\alpha = 1, \dots, N_p + n_{\text{ion}}$), we can jointly constrain them by measuring $P_{\Delta T}(\mathbf{u})$.

In Appendix A we will describe a χ^2 goodness-of-fit test for quantifying whether this parametrization is valid. The Fisher matrix for measuring p_{α} is simply

$$F_{\alpha\beta} = \sum_{\text{pixels}} \frac{1}{[\delta P_{\Delta T}(\mathbf{u})]^2} \frac{\partial P_{\Delta T}(\mathbf{u})}{\partial p_{\alpha}} \frac{\partial P_{\Delta T}(\mathbf{u})}{\partial p_{\beta}}. \quad (24)$$

This Fisher matrix $F_{\alpha\beta}$ is not block diagonal, *i.e.*, there are correlations between the cosmological and ionization parameters, reflecting the fact that both affect $\mathcal{P}_{xx}(k)$ and $\mathcal{P}_{x\delta}(k)$. The inversion of the Fisher matrix therefore leads to the degradation of the constraints of cosmological parameters. However, the total 21cm power spectrum is usually smaller in magnitude in the MID model than in the OPT model (see Eq. (4)), giving less sample variance. This means that as long as noise in a 21cm experiment dominates over sample variance, the MID model will give weaker constraints than the OPT model, because of the degeneracies. For future experiments with very low noise, however, it is possible to have the opposite situation, if the reduction in sample variance dominates over the increase in degeneracy. This does of course not mean that the MID model is more optimistic than the OPT model, merely that the OPT model is assuming an unrealistic power spectrum.

Having set up the general formalism, we now propose a parametrization specified by Eq. (18), with fiducial values of ionization parameters given in Table III. This parametrization was designed to match the results of the radiative transfer simulations in Model I of [28], and Figure 2 shows that the fit is rather good in the range $k = 0.1 - 2 \text{ Mpc}^{-1}$ to which the 21cm experiments we consider are most sensitive.

The radiative transfer simulations implemented in [28] are post processed on top of a 1024^3 N-body simulation in a box of size 186 Mpc. Three models for the reionization history are considered in [28]:

1. In Model I, all dark matter halos above m_{cool} (corresponding to the minimum mass galaxy in which the gas can cool by atomic transitions and form stars, e.g. $m_{\text{cool}} \approx 10^8 M_{\odot}$ at $z = 8$) contribute ionizing photons at a rate that is proportional to their mass.
2. In Model II, the ionizing luminosity of the sources scales as halo mass to the 5/3 power, i.e. more massive halos dominate the production of ionizing photons than in Model I.
3. In Model III, which has the same source parametrization as in Model I except for doubled luminosity, minihalos with $m > 10^5 M_{\odot}$ absorb incident ionizing photons out to their virial radius unless they are photo-evaporated (but do not contribute ionizing photons).

It appears to be a generic feature in the simulation results that the ratios of functions at large k fall off like a power law for $\mathcal{P}_{xx}(k)/\mathcal{P}_{\delta\delta}(k)$, and exponentially for $\mathcal{P}_{x\delta}(k)/\mathcal{P}_{\delta\delta}(k)$. At small k , $\mathcal{P}_{xx}(k)/\mathcal{P}_{\delta\delta}(k)$ can either increase or decrease approximately linearly as k increases, while $\mathcal{P}_{x\delta}(k)/\mathcal{P}_{\delta\delta}(k)$ is asymptotically constant. Our parametrization in Eq. (18) captures these features: at large k , $\mathcal{P}_{xx}(k)/\mathcal{P}_{\delta\delta}(k) \propto k^{-\gamma_{xx}}$ and $\mathcal{P}_{x\delta}(k)/\mathcal{P}_{\delta\delta}(k) \propto \exp(-(k R_{x\delta})^2)$; at small k , $\mathcal{P}_{xx}(k)/\mathcal{P}_{\delta\delta}(k) \propto (1 - (\gamma_{xx} \alpha_{xx} R_{xx}/2)k)$, and

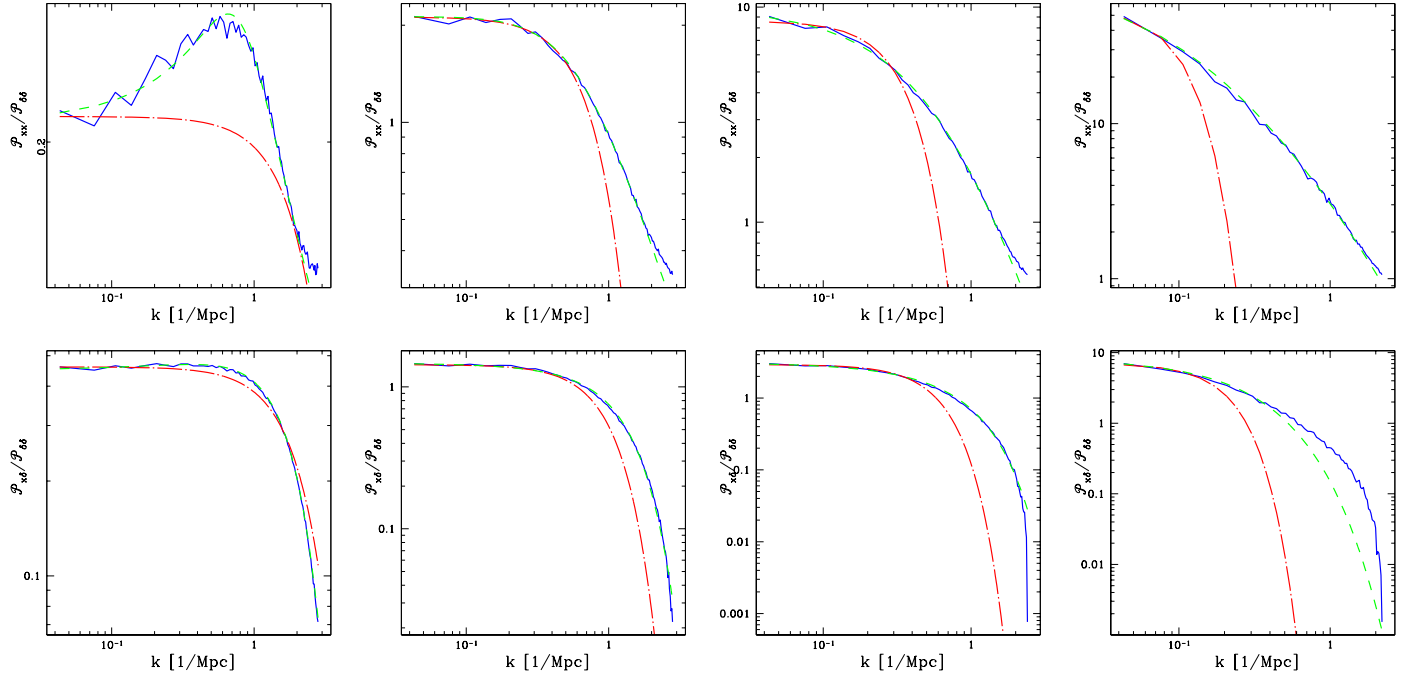


FIG. 2: Fits to the ionization power spectra at several redshifts. Solid (blue) lines are the results of the radiative transfer simulation in Model I of the McQuinn et al. paper [28]. Dashed (green) lines are fitting curves of our parametrization. Dot-dashed (red) lines are best fits using the parametrization suggested by Santos and Cooray [3]. Top panels: $\mathcal{P}_{xx}/\mathcal{P}_{\delta\delta} = P_{xx}/(\bar{x}_H^2 P_{\delta\delta})$. Bottom panels: $\mathcal{P}_{xd}/\mathcal{P}_{\delta\delta} = P_{xd}/(\bar{x}_H P_{\delta\delta})$. From left to right: $z = 9.2, 8.0, 7.5, 7.0$ ($\bar{x}_i = 0.10, 0.30, 0.50, 0.70$ respectively).

$\mathcal{P}_{x\delta}(k)/\mathcal{P}_{\delta\delta}(k) \propto (1 - \alpha_{x\delta} R_{x\delta} k)$ (both α_{xx} and $\alpha_{x\delta}$ can be either positive or negative). Figure 2 also shows that for $P_{xx}(k)$ and also for $P_{x\delta}(k)$ at large k , our parametrization further improves over the parametrization $P(k)/P_{\delta\delta} = b^2 e^{-(kR)^2}$ suggested by Santos and Cooray [3], which works well for $P_{x\delta}(k)$ at small k .

To be conservative, we discard cosmological information from $\mathcal{P}_{x\delta}(k)$ and $\mathcal{P}_{xx}(k)$ in our Fisher matrix analysis by using the fiducial power spectrum $\mathcal{P}_{\delta\delta}(k)^{(\text{fid})}$ rather than the actual one $\mathcal{P}_{\delta\delta}(k)$ in Eq. (18). This means that the derivatives of $\mathcal{P}_{x\delta}(k)$ and $\mathcal{P}_{xx}(k)$ with respect to the cosmological parameters vanish in Eq. (24). It is likely that we can do better in the future: once the relation between the ionization power spectra and the matter power spectrum can be reliably calculated either analytically or numerically, the ionization power spectra can contribute to further constraining cosmology.

In addition to the fit of Model I shown in Figure 2, we also fit our model (with different fiducial values from those listed in Table III) to the simulations using Model II and III in [28], and find that the parametrization is flexible enough to provide good fits to all three simulations, suggesting that the parametrization in Eq. (18) may be generically valid and independent of models. Note, however, that at low redshifts ($\bar{x}_i \gtrsim 0.7$), our parametrization of $\mathcal{P}_{x\delta}/\mathcal{P}_{\delta\delta}$ does not work well at large k , in that the

simulation falls off less rapidly than exponentially. This may be because when HII regions dominate the IGM, the ionized bubbles overlap in complicated patterns and correlate extremely non-linearly at small scales. This partial incompatibility indicates that our parametrization (i.e. Eq.18) is only accurate for small \bar{x}_i , i.e. before non-linear ionization patterns come into play.

In the remainder of this paper, we will adopt the values in Table III as fiducial values of the ionization parameters.

3. PESS model

By parametrizing the ionization power spectra with a small number of constants, the MID model rests on our understanding of the physics of reionization. From the point of view of a maximally cautious experimentalist, however, constraints on cosmological parameters should not depend on how well one models reionization. In this spirit, Barkana and Loeb [23, 24] proposed what we adopt as our “PESS” model for separating the physics $\mathcal{P}_{\delta\delta}(k)$ from the “gastrophysics” $\mathcal{P}_{xx}(k)$ and $\mathcal{P}_{x\delta}(k)$. Instead of assuming a specific parametrization, the PESS model makes no *a priori* assumptions about the ionization power spectra. In each k -bin that contains more

than three pixels in \mathbf{u} -space, one can in principle separate $P_{\mu^4}(k) = \mathcal{P}_{\delta\delta}(k)$ from the other two moments. The PESS model essentially only constrains cosmology from the P_{μ^4} term and therefore loses all information in P_{μ^0} and P_{μ^2} . We now set up the Fisher matrix formalism for the PESS model that takes advantage of the anisotropy in $P_{\Delta T}(\mathbf{k})$ arising from the velocity field effect. Numerical evaluations will be performed in Section III A.

The observable in 21cm tomography is the brightness temperature $T_b(\mathbf{x})$. In Fourier space, the covariance matrix between two pixels \mathbf{k}_i and \mathbf{k}_j is $\mathbf{C}_{ij} = \delta_{ij}[P_{\Delta T}(\mathbf{k}_i) + P_N(k_\perp)]$, assuming that the measurements in two different pixels are uncorrelated². The total 21cm power spectrum is $P_{\Delta T}(\mathbf{k}) = P_{\mu^0}(k) + P_{\mu^2}(k)\mu^2 + P_{\mu^4}(k)\mu^4$. For convenience, we use the shorthand notation P_A , where $P_1 \equiv P_{\mu^0}$, $P_2 \equiv P_{\mu^2}$ and $P_3 \equiv P_{\mu^4}$ and define the $a_A = 0, 2, 4$ for $A = 1, 2, 3$, respectively. Thus the power spectrum can be rewritten as $P_{\Delta T} = \sum_{A=1}^3 P_A \mu^{a_A}$. Treating $P_A(k)$ at each k -bin as parameters, the derivatives of the covariance matrix are simply $\partial \mathbf{C}_{ij} / \partial P_A(k) = \delta_{ij} \mu^{a_A}$, where $|\mathbf{k}_i|$ resides in the shell of radius k with width Δk . Since the different k -bins all decouple, the Fisher matrix for measuring the moments $P_A(k)$ is simply a separate 3×3 -matrix for each k -bin:

$$F_{AA'}(k) = \frac{1}{2} \text{tr} \left[\mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial P_A(k)} \mathbf{C}^{-1} \frac{\partial \mathbf{C}}{\partial P_{A'}(k)} \right] \\ = \sum_{\text{upper half-shell}} \frac{\mu^{a_A + a_{A'}}}{[\delta P_{\Delta T}(\mathbf{k})]^2}, \quad (25)$$

where $\delta P_{\Delta T}(\mathbf{k}) = N_c^{-1/2} [P_{\Delta T}(\mathbf{k}) + P_N(k_\perp)]$. Here $P_N(k_\perp)$ is related to $P_N(u_\perp)$ by Eq. (16). Again the sum is over the upper half of the spherical shell $k < |\mathbf{k}| < k + \Delta k$. The 1σ error of $P_3 = P_{\mu^4}$ is $\delta P_3(k) = \sqrt{F^{-1}_{33}(k)}$. Once $\mathcal{P}_{\delta\delta} = P_{\mu^4}$ is separated from other moments, $\mathcal{P}_{\delta\delta}$ can be used to constrain cosmological parameters λ_a with the Fisher matrix as given in Eq. (21).

We have hitherto discussed the anisotropy in $P_{\Delta T}(\mathbf{k})$ that arises from the velocity field effect. However, the AP-effect may further contribute to the anisotropy in that it creates a μ^6 -dependence and modifies the μ^4 term [26, 27]. The AP-effect can be distinguished from the velocity field effect since the P_{μ^6} term is unique to the AP-effect. Thus, one can constrain cosmological parameters from P_{μ^4} and P_{μ^6} [2], involving the inversion of a 4×4 matrix which loses even more information and therefore further weakens constraints. Therefore, the PESS Fisher matrix that we have derived without taking AP-effect into account can be viewed as an upper bound on how well the PESS approach can do in terms of cosmological parameter constraints. However, this maximally conservative 4×4 matrix approach may be inappropriately pessimistic, since the AP-induced clustering

anisotropy is typically very small within the observationally allowed cosmological parameter range, whereas the velocity-induced anisotropies can be of order unity.

C. Assumptions about Linearity

To avoid fitting to modes where δ_k is non-linear and physical modeling is less reliable, we impose a sharp cut-off at k_{max} and exclude all information for $k > k_{\text{max}}$. We take $k_{\text{max}} = 2 \text{ Mpc}^{-1}$ for our MID model, and investigate the k_{max} -dependence of cosmological parameter constraints in Section III B.

D. Assumptions about non-Gaussianity

Non-Gaussianity of ionization signals generically becomes important at high \bar{x}_i . Using cosmic reionization simulations with a large volume and high resolution, Lidz et al. [39] and Santos et al. [40] found non-negligible (a factor of 1.5) differences in the full power spectrum at high \bar{x}_i ($\bar{x}_i \gtrsim 0.35$). To get a rough sense of the impact of non-Gaussianity on cosmological parameter constraints, we simply model it as increasing the sample variance by a factor ξ . We thus write the total power spectrum as

$$\delta P_{\Delta T}(\mathbf{u}) = N_c^{-1/2} [\xi P_{\Delta T}(\mathbf{u}) + P_N(u_\perp)], \quad (26)$$

where ξ is the factor by which the sample variance is increased. The parameter ξ should take the value $\xi \approx 1$ (Gaussian) at epochs with low \bar{x}_i and $1 < \xi \lesssim 2$ (non-Gaussian) at high \bar{x}_i .

E. Assumptions about reionization history and redshift range

21cm tomography can probe a wide range of redshifts, as illustrated in Figure 1. However, one clearly cannot simply measure a single power spectrum for the entire volume, as the clustering evolves with cosmic time: The matter power spectrum changes gradually due to the linear growth of perturbations [37]. More importantly, the ionization power spectra vary dramatically with redshift through the epoch of reionization. We incorporate these complications by performing our analysis separately in a series of redshift slices, each chosen to be narrow enough that the matter and ionization power spectra can be approximated as constant in redshift within each slice. This dictates that for a given assumed reionization history, thinner redshift slices must be used around redshifts where \bar{x}_H varies dramatically.

In this paper, we will consider two rather opposite toy models in Section III:

- OPT: A sharp reionization that begins and finishes at one redshift (say $z \lesssim 7$).

² We ignore here a δ -function centered at the origin since 21cm experiments will not measure any $k = 0$ modes.

TABLE IV: Specifications for 21cm interferometers

| Experiment | N_{ant} | Min. base-line (m) | f.o.v. (deg ²) | A_e (m ²) at $z=6/8/12^a$ |
|------------|------------------|--------------------|----------------------------|---|
| MWA | 500 | 4 | $\pi 16^2$ | 9/14/18 |
| SKA | 7000 | 10 | $\pi 8.6^2$ | 30/50/104 |
| LOFAR | 77 | 100 | $2 \times \pi 2.4^2$ | 397/656/1369 |
| FFTT | 10^6 | 1 | 2π | 1/1/1 |

^aWe assume that the effective collecting area is proportional to λ^2 such that the sensitivity (A_e/T_{sys} in m²K⁻¹) meets the design specification.

- MID/PESS: A gradual reionization that spans a range of redshifts, assuming the ionization parameter values that fit Model I simulation of the McQuinn et al. paper [28]

For the latter scenario, the ionization fraction \bar{x}_H is not a linear function of redshift. For example, in the McQuinn et al. [28] simulation, $\bar{x}_H = 0.9, 0.7, 0.5$ and 0.3 correspond to redshifts $z = 9.2, 8.0, 7.5$ and 7.0 , respectively. For our different scenarios, we therefore adopt the redshift ranges $6.8 < z < 10$ that are divided into four redshift slices centered at the above redshifts (OPT), $6.8 < z < 8.2$ split into three bins centered at $z=7.0, 7.5$ and 8.0 (MID), $7.3 < z < 8.2$ split into two slices centered at $z = 7.5$ and 8.0 .

F. Assumptions about cosmological parameter space

Since the impact of the choice of cosmological parameter space and related degeneracies has been extensively studied in the literature, we will perform only a basic analysis of this here. We work within the context of standard inflationary cosmology with adiabatic perturbations, and parametrize cosmological models in terms of 12 parameters (see, e.g., Table 2 in [45] for explicit definitions) whose fiducial values are assumed as follows: $\Omega_k = 0$ (spatial curvature), $\Omega_\Lambda = 0.7$ (dark energy density), $\Omega_b = 0.046$ (baryon density), $h = 0.7$ (Hubble parameter $H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$), $\tau = 0.1$ (reionization optical depth), $\Omega_\nu = 0.0175$ (massive neutrino density), $n_s = 0.95$ (scalar spectral index), $A_s = 0.83$ (scalar fluctuation amplitude), $r = 0$ (tensor-to-scalar ratio), $\alpha = 0$ (running of spectral index), $n_t = 0$ (tensor spectral index) and $w = -1$ (dark energy equation of state). We will frequently use the term “vanilla” to refer to the minimal model space parametrized by $(\Omega_\Lambda, \Omega_m h^2, \Omega_b h^2, n_s, A_s, \tau)$ combined with $\bar{x}_H(z)$ and ionization parameters at all observed z -bins, setting $\Omega_k, \Omega_\nu h^2, r, \alpha, n_t$, and w fixed at their fiducial values.

G. Assumptions about Data

The MWA, LOFAR, SKA and FFTT instruments are still in their planning/design/development stages. In this paper, we adopt the key design parameters from [29] for MWA, [30] and www.skatelescope.org for SKA, www.lofar.org for LOFAR, and [11] for FFTT unless explicitly stated.

1. Interferometers

We assume that MWA will have 500 correlated $4\text{m} \times 4\text{m}$ antenna tiles, each with 16 dipoles. Each individual tile will have an effective collecting area of 14 m^2 at $z = 8$ and 18 m^2 at $z \gtrsim 12$. LOFAR will have 77 large (diameter $\sim 100 \text{ m}$) stations, each with thousands of dipole antennae such that it has the collecting area nearly 50 times larger than each antenna tile of MWA. Each station can simultaneously image N regions in the sky. We set $N = 2$ in this paper but this number may be larger for the real array. The design of SKA has not been finalized. We assume the “smaller antennae” version of SKA, in which SKA will have 7000 small antennae, much like MWA, but each panel with much larger collecting area. FFTT stands for Fast Fourier Transform Telescope, a future square kilometer array optimized for 21 cm tomography as described in [11]. Unlike the other interferometers we consider, which add in phase the dipoles in each panel or station, FFTT correlates all of its dipoles, resulting in more information. We evaluate the case where FFTT contains a million $1\text{m} \times 1\text{m}$ dipole antennae in a contiguous core subtending a square kilometer, providing a field-of-view of 2π steradians.

For all interferometers, we assume that the collecting area $A_e \propto \lambda^2$, like a simple dipole, except that A_e is saturated at $z \sim 12$ in MWA since the wavelength $\lambda = 21(1+z) \text{ cm}$ exceeds the physical radius of an MWA antenna panel. The summary of the detailed specifications adopted in this paper is listed in Table IV.

2. Configuration

The planned configurations of the above-mentioned interferometers are quite varied. However, all involve some combination of the following elements, which we will explore in our calculations:

1. A *nucleus* of radius R_0 within which the area coverage fraction is close to 100%.
2. A *core* extending from radius R_0 out to R_{in} where there coverage density drops like some power law r^{-n} .
3. An *annulus* extending from R_{in} to R_{out} where the coverage density is low but rather uniform.

In its currently planned design, the MWA will have a small nucleus, while the core density falls off as r^{-2} until a sharp cutoff at R_{in} . For LOFAR we assume 32 stations in the core, and another 32 stations in an outer annulus out to radius $R_{\text{out}} \sim 6$ km. For SKA we assume 20% in the core, and 30% in the annulus out to radius $R_{\text{out}} \sim 5$ km. We ignore the measurements from any dilute distribution of antenna panels outside R_{out} . For LOFAR and SKA, we assume a uniform distribution of antennae in the annulus, but with an inner core profile like that of the MWA, i.e., a nucleus of radius $R_0 = 285/189$ m (LOFAR/SKA) and an r^{-2} fall-off outside this compact core. We assume an azimuthally symmetric distribution of baselines in all arrays.

For an array with N_{in} antennae within R_{in} , we can define a quantity

$$R_0^{\text{max}} \equiv \sqrt{\frac{N_{\text{in}}}{\rho_0 \pi}}, \quad (27)$$

where ρ_0 is the area density of the nucleus. R_0^{max} is the maximal radius of the nucleus, corresponding to the case where there it contains all the N_{in} antennae and there is no core.

It is also convenient to parametrize the distribution of these N_{in} antennae within R_{in} by two numbers: the fraction η that are in the nucleus, and the fall-off index n of the core. It is straightforward to show that R_0 and R_{in} are related to η and n by

$$R_0 = \sqrt{\eta} R_0^{\text{max}}, \quad (28)$$

$$R_{\text{in}} = R_0 \left(\frac{2 - n(1 - \eta)}{2\eta} \right)^{\frac{1}{2-n}} \quad (29)$$

if $n \neq 2$. The analytic relation for $n = 2$ is $R_{\text{in}} = R_0 \exp[(1 - \eta)/(2\eta)]$, which can be well approximated in numerical calculation by taking $n = 2 + \epsilon$ in Eq. (29) with $\epsilon \sim 10^{-10}$.

In Section III E, we will scan almost all possible design configurations and find the optimal one for constraining cosmology. There are two independent ways to vary array configurations, as illustrated by Figure 3: by varying R_0 with R_{in} fixed, and by varying R_{in} with R_0 fixed. Contributions from antennae in the annulus are negligible compared to the core, so varying R_{out} is not interesting.

In other parts of Section III, we will assume the intermediate configuration $\eta = 0.8$ and $n = 2$ (except for FFTT which is purely in a giant core) with the planned number of antennae in the core and annulus. Note that this configuration is optimized from the currently planned design.

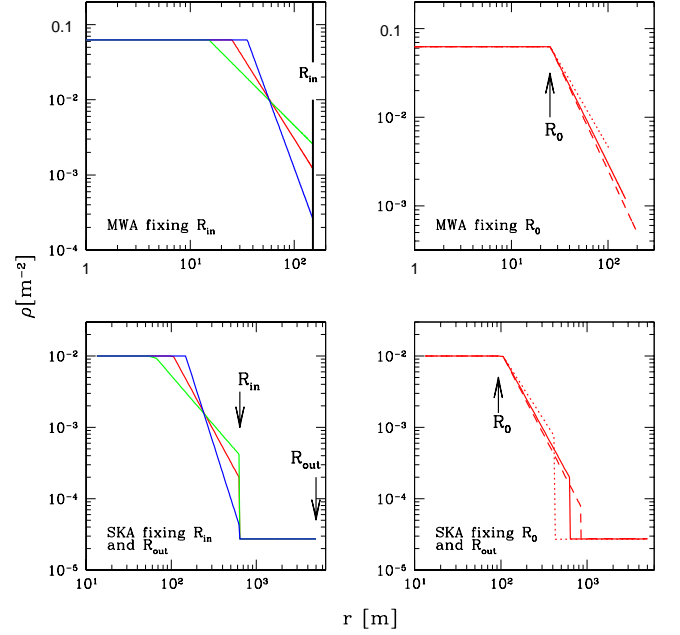


FIG. 3: Examples of array configuration changes. For MWA (upper panels), antennae are uniformly distributed inside the nucleus radius R_0 , and the density ρ falls off like a power law for $R_0 < r < R_{\text{in}}$ where R_{in} is the core radius. For SKA (lower panels) and similarly for LOFAR, there is in addition a uniform yet dilute distribution of antennae in the annulus $R_{\text{in}} < r < R_{\text{out}}$, where R_{out} is the outer annulus radius. When R_0 is decreased ($R_0 = 0.7/0.5/0.3 \times R_0^{\text{max}}$) with $R_{\text{in}} = 3.0 \times R_0^{\text{max}}$ fixed (left panels), the density in the core falls off slower (blue/red/green curves). When R_{in} is decreased ($R_{\text{in}} = 4.0/3.0/2.0 \times R_0^{\text{max}}$) with $R_0 = 0.5 \times R_0^{\text{max}}$ fixed (right panels), the density in the core also falls off less steep (dashed/solid/dotted curves).

3. Detector noise

21cm radio interferometers measure visibility \mathbf{V} . The visibility for a pair of antennae is defined as [31]

$$\mathbf{V}(u_x, u_y, \Delta f) = \int dx dy \Delta T_b(x, y, \Delta f) e^{-i(u_x x + u_y y)}, \quad (30)$$

where (u_x, u_y) are the number of wavelengths between the antennae. The hydrogen 3D map is the Fourier transform in the frequency direction $\tilde{I}(\mathbf{u}) \equiv \int d\Delta f \mathbf{V}(u_x, u_y, \Delta f) \exp(-i\Delta f u_{\parallel})$ where $\mathbf{u} = u_x \hat{e}_x + u_y \hat{e}_y + u_{\parallel} \hat{e}_z$. The detector noise covariance matrix for an interferometer is [2, 32]

$$C^N(\mathbf{u}_i, \mathbf{u}_j) = \left(\frac{\lambda^2 B T_{\text{sys}}}{A_e} \right)^2 \frac{\delta_{ij}}{B t_{\mathbf{u}_i}}, \quad (31)$$

where B is the frequency bin size, T_{sys} is system temperature, and $t_{\mathbf{u}} \approx (A_e t_0 / \lambda^2) n(u_{\perp})$ is the observation time for the visibility at $|\mathbf{u}_{\perp}| = d_A |\mathbf{k}| \sin \theta$. Here t_0 is the total

observation time, and n is the number of baselines in an observing cell.

The covariance matrix of the 21cm signal $\tilde{I}(\mathbf{u})$ is related to the power spectrum $P_{\Delta T}(\mathbf{k})$ by [2]

$$\begin{aligned} C^{SV}(\mathbf{u}_i, \mathbf{u}_j) &\equiv \langle \tilde{I}^*(\mathbf{u}_i) \tilde{I}(\mathbf{u}_j) \rangle \\ &= P_{\Delta T}(\mathbf{u}_i) \frac{\lambda^2 B}{A_e} \delta_{ij}. \end{aligned} \quad (32)$$

Therefore, the noise in the power spectrum is

$$P^N(u_\perp) = \left(\frac{\lambda^2 T_{\text{sys}}}{A_e} \right)^2 \frac{1}{t_0 n(u_\perp)}. \quad (33)$$

For all interferometers, the system temperature is dominated by sky temperature $T_{\text{sky}} \approx 60(\lambda/1\text{m})^{2.55}\text{K}$ due to synchrotron radiation in reasonably clean parts of the sky. Following [29], we set $T_{\text{sys}} = 440\text{K}$ at $z = 8$ and $T_{\text{sys}} = 690\text{K}$ at $z = 10$.

H. Assumptions about Residual Foregrounds

There have been a number of papers discussing foreground removal for 21 cm tomography (e.g. [33, 34, 35, 36] and references therein), and much work remains to be done on this important subject, as the amplitudes of residual foregrounds depend strongly depends on cleaning techniques and assumptions, and can have potentially dominate the cosmological signal. The work of Wang et al. [33] and McQuinn et al. [2] suggested that after fitting out a low-order polynomial from the frequency dependence in each pixel, the residual foregrounds were negligible for $k > 2\pi/yB$ where yB is the comoving width of a z -bin. To obtain a crude indication of the impact of residual foregrounds, there therefore we adopt the rough approximation that all data below some cutoff value k_{min} is destroyed by foregrounds while the remainder has negligible contamination. We choose $k_{\text{min}} = (1/2/4) \times \pi/yB$ for the OPT/MID/PESS scenarios, and also explore wider ranges below.

III. RESULTS AND DISCUSSION

In this section, we numerically evaluate how the accuracy of cosmological parameter constraints depend on the various assumptions listed above. Where possible, we attempt to provide intuition for these dependences with simple analytical approximations. In most cases, we explore the dependence on one assumption at a time by evaluating the PESS, MID and OPT scenario for this assumption while keeping all other assumptions fixed to the baseline MID scenario.

A. Varying ionization power spectrum modeling and reionization histories

1. Basic results

We start by testing assumptions in the ionization power modeling of P_{xx} and $P_{\text{x}\delta}$. In Table V we show the accuracy with which the 21cm power spectrum can place constraints on the cosmological parameters from three z -bins ranging from $z = 6.8 - 8.2$. We fix the assumptions concerning k_{max} , the foreground removal, and the array layout and specifications, but vary the sophistication with which we model the ionization power.

Our results agree with those of previous studies [1, 2], i.e. 21cm data alone (except for the optimized FFTT) cannot place constraints comparable with those from Planck CMB data. However, if 21cm data are combined with CMB data, the parameter degeneracies can be broken, yielding stringent constraints on Ω_k , m_ν and α . For example, in the OPT model, from LOFAR/MWA/SKA/FFTT combined with Planck, the curvature density Ω_k can be measured 5/3/15/78 times better, to a precision $\Delta\Omega_k = 0.005/0.008/0.002/0.0003$, the neutrino mass m_ν can be constrained 4/9/14/74 times better to accuracy $\Delta m_\nu = 0.06/0.03/0.02/0.003$, and running of the scalar spectral index α can be done 1/2/4/28 times better, to $\Delta\alpha = 0.002/0.001/0.0006/0.0001$. The more realistic MID model yields weaker yet still impressive constraints: from SKA/FFTT combined with Planck, Ω_k can be measured 6/109 times better, to $\Delta\Omega_k = 0.004/0.0002$, m_ν 4/35 times better, to $\Delta m_\nu = 0.06/0.007$, and α 1/15 times better, to $\Delta\alpha = 0.002/0.0002$. The improved measurements of Ω_k and α enable further precision tests of inflation, since generically Ω_k is predicted to vanish down to the 10^{-5} level, while the simplest inflation models (with a single slow-rolling scalar field) predict $\alpha \sim (1 - n_s)^2 \sim 10^{-3}$. For example, the inflaton potential $V(\phi) \propto \phi^2$ predicts $\alpha \approx -0.0007$, while $V(\phi) \propto \phi^4$ predicts $\alpha = 0.008$. In addition, 21cm data combined with CMB data from Planck can make accurate measurements in the mean neutral fraction $\bar{x}_H(z)$ at separate redshifts, outlining the full path of reionization, e.g. at the $\Delta\bar{x}_H(z) \sim 0.01/0.003$ level from SKA/FFTT data combined with Planck data.

2. OPT and MID models

For most 21cm experiments, the OPT model yields stronger constraints than the MID model. The reason is as follows. By assuming $P_{\text{xx}} = P_{\text{x}\delta} = 0$, there are essentially no neutral fraction fluctuations in the OPT model. This means that this model is an ideal model in which the 21cm power spectrum encodes cosmological information per se, since $P_{\Delta T}(\mathbf{k}) \propto \mathcal{P}_{\delta\delta}(k)$ at each pixel in the Fourier space. In the more realistic MID model, however, the nuisance ionization parameters has correla-

TABLE V: How cosmological constraints depend on the ionization power spectrum modeling and reionization history. We assume observations of 4000 hours on two places in the sky in the range of $z = 6.8 - 8.2$ that is divided into three z -bins centered at $z = 7.0, 7.5$ and 8.0 respectively, $k_{\max} = 2\text{Mpc}^{-1}$, $k_{\min} = 2\pi/yB$ and a quasi-giant core configuration (except for FFTT that is a giant core). 1σ errors of ionization parameters in the MID model, marginalized over other vanilla parameters, are listed separately in Table VI.

| | Model | Vanilla Alone | | | | | | | | | $\Delta\Omega_k$ | Δm_ν [eV] | $\Delta\alpha$ |
|--------|-------|------------------------|---------------------------|---------------------------|--------------|-----------------|--------------|--------------------------|------------------------|------------------------|------------------|---------------------|----------------|
| | | $\Delta\Omega_\Lambda$ | $\Delta\ln(\Omega_m h^2)$ | $\Delta\ln(\Omega_b h^2)$ | Δn_s | $\Delta\ln A_s$ | $\Delta\tau$ | $\Delta\bar{x}_H(7.0)^a$ | $\Delta\bar{x}_H(7.5)$ | $\Delta\bar{x}_H(8.0)$ | | | |
| LOFAR | OPT | 0.025 | 0.27 | 0.44 | 0.063 | 0.89 | ... | ... | ... | ... | 0.14 | 0.87 | 0.027 |
| | MID | 0.13 | 0.083 | 0.15 | 0.36 | 0.80 | ... | ... | ... | ... | 0.35 | 12 | 0.17 |
| MWA | OPT | 0.046 | 0.11 | 0.19 | 0.022 | 0.37 | ... | ... | ... | ... | 0.056 | 0.38 | 0.013 |
| | MID | 0.22 | 0.017 | 0.029 | 0.097 | 0.76 | ... | ... | ... | ... | 0.13 | 9.6 | 0.074 |
| SKA | OPT | 0.0038 | 0.044 | 0.083 | 0.0079 | 0.16 | ... | ... | ... | ... | 0.023 | 0.12 | 0.0040 |
| | MID | 0.014 | 0.0049 | 0.0081 | 0.012 | 0.037 | ... | ... | ... | ... | 0.043 | 0.36 | 0.0060 |
| FFTT | OPT | 0.00015 | 0.0032 | 0.0083 | 0.00040 | 0.015 | ... | ... | ... | ... | 0.00098 | 0.011 | 0.00034 |
| | MID | 0.00041 | 0.00038 | 0.00062 | 0.00036 | 0.0013 | ... | ... | ... | ... | 0.0037 | 0.0078 | 0.00017 |
| | PESS | 1.1 | 0.017 | 0.037 | 0.010 | 0.19 | ... | ... | ... | ... | ... | 0.20 | 0.0058 |
| Planck | | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | ... | ... | 0.025 | 0.23 | 0.0026 |
| +LOFAR | OPT | 0.0066 | 0.0077 | 0.0058 | 0.0031 | 0.0088 | 0.0043 | 0.0077 | 0.0084 | 0.0093 | 0.0051 | 0.060 | 0.0022 |
| | MID | 0.0070 | 0.0081 | 0.0059 | 0.0032 | 0.0088 | 0.0043 | 0.18 | 0.26 | 0.23 | 0.018 | 0.22 | 0.0026 |
| | PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | 0.54 | 0.31 | 0.24 | 0.025 | 0.23 | 0.0026 |
| +MWA | OPT | 0.0067 | 0.0079 | 0.0057 | 0.0031 | 0.0088 | 0.0043 | 0.0065 | 0.0067 | 0.0069 | 0.0079 | 0.027 | 0.0014 |
| | MID | 0.0061 | 0.0070 | 0.0056 | 0.0030 | 0.0087 | 0.0043 | 0.32 | 0.22 | 0.29 | 0.021 | 0.19 | 0.0026 |
| | PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | 3.8 | 0.87 | 0.53 | 0.025 | 0.23 | 0.0026 |
| +SKA | OPT | 0.0031 | 0.0038 | 0.0046 | 0.0013 | 0.0087 | 0.0042 | 0.0060 | 0.0060 | 0.0060 | 0.0017 | 0.017 | 0.00064 |
| | MID | 0.0036 | 0.0040 | 0.0044 | 0.0025 | 0.0087 | 0.0043 | 0.0094 | 0.014 | 0.011 | 0.0039 | 0.056 | 0.0022 |
| | PESS | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | 0.061 | 0.024 | 0.012 | 0.025 | 0.21 | 0.0026 |
| +FFTT | OPT | 0.00015 | 0.0015 | 0.0036 | 0.00021 | 0.0087 | 0.0042 | 0.0056 | 0.0056 | 0.0056 | 0.00032 | 0.0031 | 0.000094 |
| | MID | 0.00038 | 0.00034 | 0.00059 | 0.00033 | 0.0086 | 0.0042 | 0.0013 | 0.0022 | 0.0031 | 0.00023 | 0.0066 | 0.00017 |
| | PESS | 0.0055 | 0.0064 | 0.0051 | 0.0030 | 0.0087 | 0.0043 | 0.0024 | 0.0029 | 0.0040 | 0.025 | 0.020 | 0.0010 |

^a $\bar{x}_H(z)$ denotes the mean neutral fraction at the central redshift z . $\bar{x}_H(z)$'s and A_s are completely degenerate from the 21cm measurement alone. For this reason, the errors shown for $\ln A_s$ from 21cm data alone is really not marginalized over $\bar{x}_H(z)$'s.

TABLE VI: 1σ marginalized errors for the ionization parameters in the MID model. Assumptions are made the same as in Table V. R_{xx} and $R_{x\delta}$ are in units of Mpc and other parameters are unitless.

| z | Δb_{xx}^2 | ΔR_{xx} | $\Delta\alpha_{xx}$ | $\Delta\gamma_{xx}$ | $\Delta b_{x\delta}^2$ | $\Delta R_{x\delta}$ | $\Delta\alpha_{x\delta}$ |
|-----------|-------------------|-----------------|---------------------|---------------------|------------------------|----------------------|--------------------------|
| Values | 77. | 3.0 | 4.5 | 2.05 | 8.2 | 0.143 | 28. |
| 7.0 LOFAR | 94 | 140 | 130 | 27 | 5.1 | 49 | 9600 |
| MWA | 20 | 43 | 43 | 8.3 | 2.6 | 16 | 3200 |
| SKA | 9.1 | 9.8 | 8.7 | 2.0 | 0.49 | 2.6 | 520 |
| FFTT | 0.59 | 0.47 | 0.39 | 0.098 | 0.027 | 0.088 | 17 |
| Values | 9.9 | 1.3 | 1.6 | 2.3 | 3.1 | 0.58 | 2. |
| 7.5 LOFAR | 2.2 | 55 | 18 | 73 | 1.4 | 5.7 | 24 |
| MWA | 4.3 | 16 | 4.9 | 22 | 1.8 | 1.8 | 8.1 |
| SKA | 0.18 | 1.7 | 0.71 | 2.1 | 0.076 | 0.17 | 0.78 |
| FFTT | 0.0072 | 0.027 | 0.015 | 0.030 | 0.0023 | 0.0021 | 0.012 |
| Values | 2.12 | 1.63 | -0.1 | 1.35 | 1.47 | 0.62 | 0.46 |
| 8.0 LOFAR | 1.6 | 20 | 2.1 | 34 | 1.2 | 3.4 | 6.9 |
| MWA | 2.7 | 13 | 4.2 | 24 | 1.5 | 1.6 | 2.8 |
| SKA | 0.085 | 0.60 | 0.090 | 0.90 | 0.057 | 0.095 | 0.24 |
| FFTT | 0.0017 | 0.013 | 0.0026 | 0.017 | 0.0013 | 0.0014 | 0.0030 |

tions with cosmological parameters. Mathematically, the inversion of a correlated matrix multiplies each error by a degradation factor.

An exception is the FFTT, where the situation is reversed. As mentioned in Section II B 2, the sample variance $P_{\Delta T}$ in the MID model is smaller than that in the

OPT model because of two reasons: (i) the MID model assumes non-zero P_{xx} and $P_{x\delta}$, and $P_{x\delta}$ has negative contribution to the total power spectrum (see Eqs.6 and 7); (ii) the OPT model assumes $\bar{x}_H = 1$, but \bar{x}_H takes realistic values (less than 1) in the MID model, decreasing the overall amplitude. In a signal-dominated experiment, reduced sample variance can be more important than the degradation from correlations.

3. PESS model

Our results show that even combined with CMB data from Planck, the 21cm data using the PESS model cannot significantly improve constraints. There are two reasons for this failure. Firstly, the PESS model essentially uses only $P_{\mu^4}(k)$ to constrain cosmology, by marginalizing over P_{μ^0} and P_{μ^2} . This loses a great deal of cosmological information in the contaminated P_{μ^0} and P_{μ^2} , in contrast to the situation in the OPT and MID models. Secondly, to effectively separate $P_{\mu^4}(k)$ from other two moments, the available Fourier pixels should span a large range in μ . Figure 4 shows that in MWA and FFTT, the data set is a thin cylinder instead of a sphere. The limitation in μ -range will give large degradation factors during the inversion of Fisher matrix. (In the limit that there is

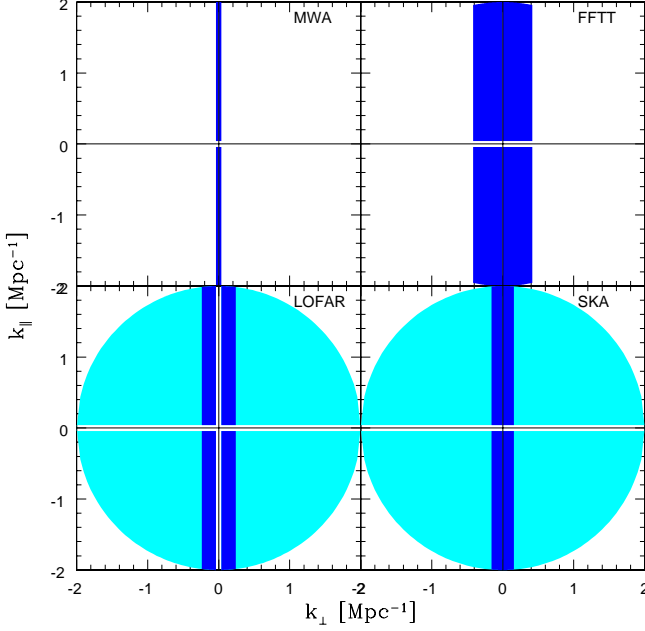


FIG. 4: Available $(k_{\perp}, k_{\parallel})$ pixels from MWA (upper left), FFTT (upper right), LOFAR (lower left) and SKA (lower right), evaluated at $z = 8$. The blue/grey regions can be measured with good signal-to-noise from the nucleus and core of an array, while the cyan/light-grey regions are measured only with the annulus and have so poor signal-to-noise that they hardly contribute to cosmological parameter constraints.

only one μ for each shell, then the Fisher matrix is singular and the degradation factor is infinite.) These two factors work together with the noise level to shrink the useful k -modes into a rather narrow range: as shown in Figure 5, $\Delta\mathcal{P}_{\delta\delta} < \mathcal{P}_{\delta\delta}$ only for $k = 0.09 - 0.4 \text{ Mpc}^{-1}$ in SKA, $k = 0.07 - 1 \text{ Mpc}^{-1}$ in FFTT and over zero modes in LOFAR and MWA.

B. Varying k_{max}

We test how varying k_{max} affects constraints in this section. The cutoff k_{max} depends on the scale at which non-linear physics, e.g. the non-linear clustering of density perturbations or the irregularities of ionized bubbles, enter the power spectrum. It is illustrated in the right panel of Figure 6 that generically cosmological constraints asymptotically approach a value as k_{max} increases above $\sim 2 \text{ Mpc}^{-1}$ (this typical scale can be larger for cosmic variance limited experiments such as FFTT). Not much cosmological information is garnered from these high- k modes because detector noise becomes increasingly important with k . The upshot is that the accuracy only weakly depends on k_{max} .

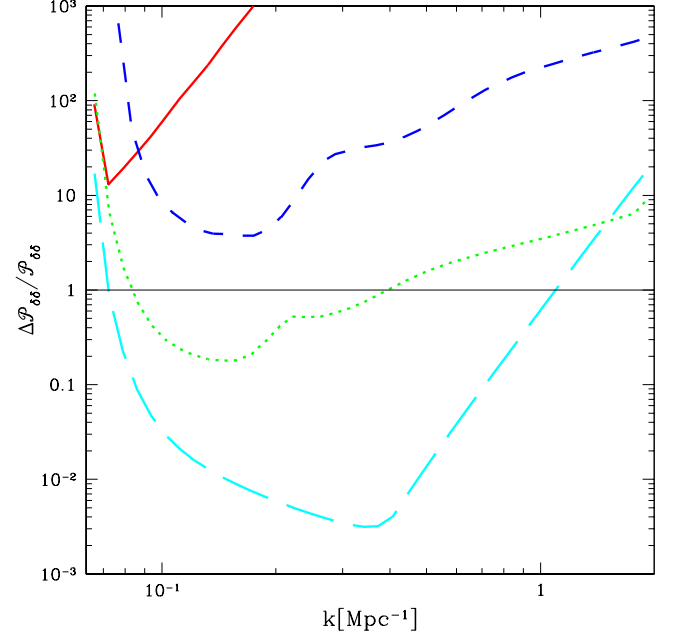


FIG. 5: Relative 1σ error for measuring $\mathcal{P}_{\delta\delta}(k)$ with the PESS model by observing a 6MHz band that is centered at $z = 8$ with MWA (red/solid), LOFAR (blue/short-dashed), SKA (green/dotted) and FFTT (cyan/long-dashed). The step size is $\Delta \ln k \approx 0.10$.

C. Varying the non-Gaussianity parameter ξ

Table II shows the effect of changing the non-Gaussianity parameter ξ in Section IID from the $\xi = 1$ (Gaussian) case to $\xi = 2$ in the PESS scenario, along with changing other assumptions. However, there is no need to perform extensive numerical investigation of the impact of ξ , since it is readily estimated analytically. Because 1σ error Δp_i in cosmological parameters is $\sqrt{(F^{-1})_{ii}}$, it follows directly from Eq. 26 that Δp does not appreciably depend on ξ for noise dominated experiments like MWA and LOFAR, whereas $\Delta p \propto \xi^\sigma$ with $\sigma \lesssim 1$ for (nearly) signal dominated experiments like SKA and FFTT. Compared with the other effects that we discuss in this section, this (no more than linear) dependence on the non-Gaussianity parameter ξ is not among the most important factors.

D. Varying redshift ranges

We now test how accuracies depend on the redshift ranges. In Table VII (OPT model) and VIII (MID model), we consider the optimistic/middle/pessimistic ranges, $z = 6.8 - 10 / 6.8 - 8.2 / 7.3 - 8.2$ which is divided by $n_z = 4/3/2$ z -bins. The results show that, from 21cm data alone, the constraints from the extreme ranges differ significantly (a factor of 5 for $\Delta\Omega_k$). Therefore, the

TABLE VII: How cosmological constraints depend on the redshift range in OPT model. Same assumptions as in Table V but for different redshift ranges and assume only OPT model.

| | | Vanilla Alone | | | | | | | | | | | | |
|---------|---------|------------------------|---------------------------|---------------------------|--------------|-----------------|--------------|------------------------|------------------------|------------------------|------------------------|------------------|---------------------|----------------|
| z range | | $\Delta\Omega_\Lambda$ | $\Delta\ln(\Omega_m h^2)$ | $\Delta\ln(\Omega_b h^2)$ | Δn_s | $\Delta\ln A_s$ | $\Delta\tau$ | $\Delta\bar{x}_H(7.0)$ | $\Delta\bar{x}_H(7.5)$ | $\Delta\bar{x}_H(8.0)$ | $\Delta\bar{x}_H(9.2)$ | $\Delta\Omega_k$ | Δm_ν [eV] | $\Delta\alpha$ |
| LOFAR | 6.8-10 | 0.021 | 0.20 | 0.34 | 0.049 | 0.67 | ... | ... | ... | ... | ... | 0.086 | 0.75 | 0.023 |
| | 6.8-8.2 | 0.025 | 0.27 | 0.44 | 0.063 | 0.89 | ... | ... | ... | ... | ... | 0.14 | 0.87 | 0.027 |
| | 7.3-8.2 | 0.036 | 0.38 | 0.61 | 0.090 | 1.2 | ... | ... | ... | ... | ... | 0.24 | 1.3 | 0.038 |
| MWA | 6.8-10 | 0.037 | 0.072 | 0.14 | 0.016 | 0.25 | ... | ... | ... | ... | ... | 0.031 | 0.31 | 0.011 |
| | 6.8-8.2 | 0.046 | 0.11 | 0.19 | 0.022 | 0.37 | ... | ... | ... | ... | ... | 0.056 | 0.38 | 0.013 |
| | 7.3-8.2 | 0.070 | 0.15 | 0.27 | 0.032 | 0.51 | ... | ... | ... | ... | ... | 0.097 | 0.53 | 0.018 |
| SKA | 6.8-10 | 0.0032 | 0.031 | 0.061 | 0.0058 | 0.12 | ... | ... | ... | ... | ... | 0.012 | 0.096 | 0.0032 |
| | 6.8-8.2 | 0.0038 | 0.044 | 0.083 | 0.0079 | 0.16 | ... | ... | ... | ... | ... | 0.023 | 0.12 | 0.0040 |
| | 7.3-8.2 | 0.0053 | 0.059 | 0.11 | 0.011 | 0.21 | ... | ... | ... | ... | ... | 0.042 | 0.17 | 0.0054 |
| FFTT | 6.8-10 | 0.00012 | 0.0023 | 0.0058 | 0.00030 | 0.011 | ... | ... | ... | ... | ... | 0.00045 | 0.0073 | 0.00023 |
| | 6.8-8.2 | 0.00015 | 0.0032 | 0.0083 | 0.00040 | 0.015 | ... | ... | ... | ... | ... | 0.00098 | 0.011 | 0.00034 |
| | 7.3-8.2 | 0.00021 | 0.0042 | 0.011 | 0.00052 | 0.019 | ... | ... | ... | ... | ... | 0.0021 | 0.014 | 0.00043 |
| Planck | | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | ... | ... | ... | 0.025 | 0.23 | 0.0026 |
| +LOFAR | 6.8-10 | 0.0065 | 0.0076 | 0.0057 | 0.0031 | 0.0088 | 0.0043 | 0.0077 | 0.0084 | 0.0082 | 0.0090 | 0.0046 | 0.051 | 0.0021 |
| | 6.8-8.2 | 0.0066 | 0.0077 | 0.0058 | 0.0031 | 0.0088 | 0.0043 | 0.0077 | 0.0084 | 0.0093 | ... | 0.0051 | 0.060 | 0.0022 |
| | 7.3-8.2 | 0.0068 | 0.0079 | 0.0058 | 0.0032 | 0.0088 | 0.0043 | ... | 0.0085 | 0.0093 | ... | 0.0072 | 0.081 | 0.0024 |
| +MWA | 6.8-10 | 0.0065 | 0.0076 | 0.0056 | 0.0031 | 0.0088 | 0.0043 | 0.0065 | 0.0067 | 0.0066 | 0.0067 | 0.0066 | 0.023 | 0.0013 |
| | 6.8-8.2 | 0.0067 | 0.0079 | 0.0057 | 0.0031 | 0.0088 | 0.0043 | 0.0065 | 0.0067 | 0.0069 | ... | 0.0079 | 0.027 | 0.0014 |
| | 7.3-8.2 | 0.0068 | 0.0080 | 0.0058 | 0.0032 | 0.0088 | 0.0043 | ... | 0.0067 | 0.0069 | ... | 0.011 | 0.036 | 0.0017 |
| +SKA | 6.8-10 | 0.0027 | 0.0035 | 0.0045 | 0.0012 | 0.0087 | 0.0042 | 0.0060 | 0.0060 | 0.0060 | 0.0060 | 0.0016 | 0.015 | 0.00061 |
| | 6.8-8.2 | 0.0031 | 0.0038 | 0.0046 | 0.0013 | 0.0087 | 0.0042 | 0.0060 | 0.0060 | 0.0060 | ... | 0.0017 | 0.017 | 0.00064 |
| | 7.3-8.2 | 0.0039 | 0.0047 | 0.0049 | 0.0017 | 0.0087 | 0.0042 | ... | 0.0060 | 0.0060 | ... | 0.0020 | 0.019 | 0.00075 |
| +FFTT | 6.8-10 | 0.00013 | 0.0014 | 0.0033 | 0.00019 | 0.0087 | 0.0042 | 0.0054 | 0.0054 | 0.0054 | 0.0054 | 0.00026 | 0.0025 | 0.000078 |
| | 6.8-8.2 | 0.00015 | 0.0015 | 0.0036 | 0.00021 | 0.0087 | 0.0042 | 0.0056 | 0.0056 | 0.0056 | ... | 0.00032 | 0.0031 | 0.000094 |
| | 7.3-8.2 | 0.00020 | 0.0016 | 0.0038 | 0.00023 | 0.0087 | 0.0042 | ... | 0.0057 | 0.0057 | ... | 0.00040 | 0.0038 | 0.00011 |

TABLE VIII: How cosmological constraints depend on the redshift range in MID model. Same assumptions as in Table V but for different redshift ranges and assume only MID model.

| Vanilla Alone | | | | | | | | | | | | | | |
|---------------|---------|------------------------|---------------------------|---------------------------|--------------|-----------------|--------------|------------------------|------------------------|------------------------|------------------------|------------------|---------------------|----------------|
| | z range | $\Delta\Omega_\Lambda$ | $\Delta\ln(\Omega_m h^2)$ | $\Delta\ln(\Omega_b h^2)$ | Δn_s | $\Delta\ln A_s$ | $\Delta\tau$ | $\Delta\bar{x}_H(7.0)$ | $\Delta\bar{x}_H(7.5)$ | $\Delta\bar{x}_H(8.0)$ | $\Delta\bar{x}_H(9.2)$ | $\Delta\Omega_k$ | Δm_ν [eV] | $\Delta\alpha$ |
| LOFAR | 6.8-10 | 0.090 | 0.055 | 0.093 | 0.18 | 0.43 | ... | ... | ... | ... | ... | 0.22 | 5.7 | 0.080 |
| | 6.8-8.2 | 0.13 | 0.083 | 0.15 | 0.36 | 0.80 | ... | ... | ... | ... | ... | 0.35 | 12 | 0.17 |
| | 7.3-8.2 | 0.21 | 0.099 | 0.15 | 0.42 | 0.81 | ... | ... | ... | ... | ... | 0.62 | 15 | 0.18 |
| MWA | 6.8-10 | 0.15 | 0.012 | 0.020 | 0.031 | 0.46 | ... | ... | ... | ... | ... | 0.092 | 4.4 | 0.025 |
| | 6.8-8.2 | 0.22 | 0.017 | 0.029 | 0.097 | 0.76 | ... | ... | ... | ... | ... | 0.13 | 9.6 | 0.074 |
| | 7.3-8.2 | 0.40 | 0.018 | 0.030 | 0.099 | 1.0 | ... | ... | ... | ... | ... | 0.32 | 18 | 0.083 |
| SKA | 6.8-10 | 0.010 | 0.0031 | 0.0056 | 0.0073 | 0.023 | ... | ... | ... | ... | ... | 0.031 | 0.23 | 0.0032 |
| | 6.8-8.2 | 0.014 | 0.0049 | 0.0081 | 0.012 | 0.037 | ... | ... | ... | ... | ... | 0.043 | 0.36 | 0.0060 |
| | 7.3-8.2 | 0.018 | 0.0050 | 0.0081 | 0.013 | 0.039 | ... | ... | ... | ... | ... | 0.072 | 0.41 | 0.0063 |
| FFTT | 6.8-10 | 0.00029 | 0.00021 | 0.00043 | 0.00025 | 0.00097 | ... | ... | ... | ... | ... | 0.0020 | 0.0055 | 0.00011 |
| | 6.8-8.2 | 0.00041 | 0.00038 | 0.00062 | 0.00036 | 0.0013 | ... | ... | ... | ... | ... | 0.0037 | 0.0078 | 0.00017 |
| | 7.3-8.2 | 0.00050 | 0.00039 | 0.00062 | 0.00037 | 0.0013 | ... | ... | ... | ... | ... | 0.0058 | 0.0083 | 0.00018 |
| Planck | | 0.0070 | 0.0081 | 0.0059 | 0.0033 | 0.0088 | 0.0043 | ... | ... | ... | ... | 0.025 | 0.23 | 0.0026 |
| +LOFAR | 6.8-10 | 0.0069 | 0.0080 | 0.0058 | 0.0032 | 0.0088 | 0.0043 | 0.18 | 0.26 | 0.15 | 0.23 | 0.017 | 0.22 | 0.0026 |
| | 6.8-8.2 | 0.0070 | 0.0081 | 0.0059 | 0.0032 | 0.0088 | 0.0043 | 0.18 | 0.26 | 0.23 | ... | 0.018 | 0.22 | 0.0026 |
| | 7.3-8.2 | 0.0070 | 0.0081 | 0.0059 | 0.0032 | 0.0088 | 0.0043 | ... | 0.27 | 0.23 | ... | 0.023 | 0.22 | 0.0026 |
| +MWA | 6.8-10 | 0.0056 | 0.0065 | 0.0054 | 0.0029 | 0.0087 | 0.0043 | 0.32 | 0.22 | 0.091 | 0.36 | 0.020 | 0.11 | 0.0025 |
| | 6.8-8.2 | 0.0061 | 0.0070 | 0.0056 | 0.0030 | 0.0087 | 0.0043 | 0.32 | 0.22 | 0.29 | ... | 0.021 | 0.19 | 0.0026 |
| | 7.3-8.2 | 0.0061 | 0.0071 | 0.0056 | 0.0030 | 0.0087 | 0.0043 | ... | 0.25 | 0.29 | ... | 0.024 | 0.19 | 0.0026 |
| +SKA | 6.8-10 | 0.0025 | 0.0027 | 0.0038 | 0.0023 | 0.0087 | 0.0042 | 0.0094 | 0.014 | 0.0075 | 0.024 | 0.0032 | 0.033 | 0.0020 |
| | 6.8-8.2 | 0.0036 | 0.0040 | 0.0044 | 0.0025 | 0.0087 | 0.0043 | 0.0094 | 0.014 | 0.011 | ... | 0.0039 | 0.056 | 0.0022 |
| | 7.3-8.2 | 0.0036 | 0.0041 | 0.0044 | 0.0025 | 0.0087 | 0.0043 | ... | 0.015 | 0.011 | ... | 0.0053 | 0.056 | 0.0023 |
| +FFTT | 6.8-10 | 0.00033 | 0.00021 | 0.00043 | 0.00024 | 0.0086 | 0.0042 | 0.0013 | 0.0022 | 0.0030 | 0.0040 | 0.00020 | 0.0052 | 0.00011 |
| | 6.8-8.2 | 0.00038 | 0.00034 | 0.00059 | 0.00033 | 0.0086 | 0.0042 | 0.0013 | 0.0022 | 0.0031 | ... | 0.00023 | 0.0066 | 0.00017 |
| | 7.3-8.2 | 0.00041 | 0.00035 | 0.00059 | 0.00033 | 0.0086 | 0.0042 | ... | 0.0022 | 0.0031 | ... | 0.00024 | 0.0070 | 0.00017 |

sensitivity of a 21cm telescope depends strongly on the frequency range over which it can observe the signal.

E. Optimal configuration: varying array layout

In this section we first investigate how array layout affects the sensitivity to cosmological parameters. Next, we investigate the optimal array configuration for fixed antennae number. Our parametrization of the array con-

figuration is discussed in Section II G 2.

We map the constraint in m_ν on the R_0 - R_{in} plane in Figure 7 (OPT model) and Figure 8 (MID model). R_0 is the radius of the compact core, and R_{in} the radius of inner core, both in the unit of $R_0^{\text{max}} \equiv \sqrt{N_{\text{in}}/\rho_0\pi}$. Note that if $R_0 = R_0^{\text{max}}$, then $R_{\text{in}} = R_0^{\text{max}}$ — this is the case of a “giant core”, in which all antennae are compactly laid down with a physical covering fraction close to unity, and is represented by the x -axis in the R_0 - R_{in} plane (the value of R_0 is meaningless if $R_{\text{in}} = R_0^{\text{max}}$). In

TABLE IX: Optimal configuration for various 21cm interferometer arrays. Same assumptions as in Table V but for different array layout. $R_{\text{in}}^{\text{prop}}$ is the previously proposed inner core radius. η is the ratio of the number of antennae in the nucleus to the total number inside the core. n is the fall-off index by which $\rho \propto r^{-n}$ outside the nucleus.

| | Experiment | R_0^{max} (m) | $R_0 (\times R_0^{\text{max}})$ | $R_{\text{in}} (\times R_0^{\text{max}})$ | $R_{\text{in}}^{\text{prop}}$ (m) ^a | η | n | Comments |
|-----|------------|------------------------|---------------------------------|---|--|--------|------|---|
| OPT | LOFAR | 319 | 0.84 | 1.28 | 1000 | 0.71 | 6.0 | Almost a giant core |
| | MWA | 50 | 0.64 | 2.41 | 750 | 0.41 | 3.0 | Close to a giant core |
| | SKA | 211 | 0.30 | 1.56 | 1000 | 0.09 | 0.83 | Almost a giant core |
| MID | LOFAR | 319 | 0.84 | 1.28 | 1000 | 0.71 | 6.0 | Almost a giant core |
| | MWA | 50 | 0.45 | 10 | 750 | 0.20 | 2.3 | Both a large nucleus and a wide-spread core |
| | SKA | 211 | 0.68 | 1.57 | 1000 | 0.46 | 2.9 | Almost a giant core |

^aNote that in LOFAR and SKA there is an annulus with the outer radius 6 km and 5 km respectively. So for them R_{in} is not the size of total array.

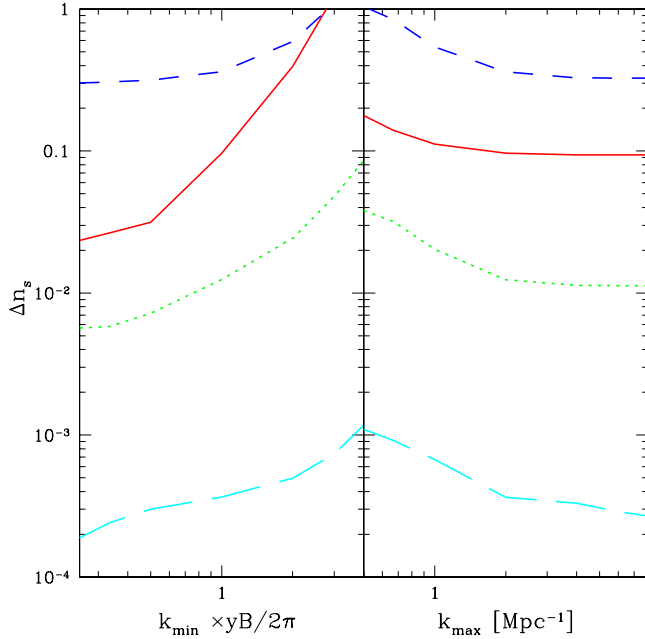


FIG. 6: How cosmological constraints Δn_s depend on k_{min} (left panel) and k_{max} (right panel) in the MID model with the 21cm experiments MWA (red/solid), LOFAR (blue/short-dashed), SKA (green/dotted) and FFTT (cyan/long-dashed). We plot Δn_s in this example because it has the strongest dependence on k_{min} and k_{max} of all cosmological parameters. The quantity $2\pi/yB$ varies in redshift, so as the horizontal axis of the left panel, we use the overall scale $\kappa_{\text{min}} \equiv k_{\text{min}} \times (yB/2\pi)$ which is equal for all z -bins,

Table IX, we list the optimal configuration that is indicated by Figure 7 and 8. The compactness of an array is represented by $R_{\text{in}}/R_0^{\text{max}}$, since R_0^{max} is the minimum of R_{in} . In comparison, R_0/R_0^{max} does not indicate the compactness, since a slow fall-off configuration with a small R_0 is effectively very close to a giant core. Rather, R_0 is a transition point from a flat compact core to the fall-off region. Note that we have three configuration parameters R_0 , R_{in} and R_{out} . We find the annulus for SKA and LOFAR to make almost no difference to the cosmological

constraints, and therefore focus on how to optimize only the remaining two parameters R_0 and R_{in} .

Table IX shows that the optimal layout for OPT model is close to a giant core, with the inner core much smaller than the previously proposed. For MID model, LOFAR and SKA still favors the quasi-giant-core layout, but MWA favors a large core whose radius is about the size that was previously proposed. The accuracies in m_ν varies in the OPT model by a factor of 3 for LOFAR, 1.4-1.5 for MWA and SKA, and in the MID model by a factor of 3 for LOFAR, 1.3 for MWA and 2.2 for SKA. This means that an optimal configuration can improve the constraints by a factor up to 3 in noise dominated experiments, and up to 2 in signal dominated experiments.

The plots have three interesting features. First, the configuration of a quasi-giant core is generically favored. The reason for this is that the noise on the temperature in an observing cell with u_\perp is inversely proportional to the square root of the number of baselines that probe this u_\perp . A compact array increases the number of baselines that probe small u_\perp , reducing the overall noise level on these modes. Second, a couple of the upcoming 21 cm experiments favor the configuration that is close but not identical to a giant core. The reason for this is because arrays become sample variance limited once they have a certain number of baselines that probe a given u_\perp . A simple estimate on the signal-to-noise ratio for a compact MWA shows that on average $\mathcal{P}_{\delta\delta}/\bar{P}_N \approx 5$ at the $k \sim 0.1 \text{ Mpc}^{-1}$ and $\mathcal{P}_{\delta\delta}/\bar{P}_N \approx 1/40$ at the $k \sim 0.7 \text{ Mpc}^{-1}$. Although moving more antennae to the center can increase the signal-to-noise, the error cannot be reduced as much if modes are already dominated by signal. Third, in the MID model, MWA favors a less compact core. This fact is due to the mixing between cosmological and ionization parameters. Remember that the off-diagonal elements in the Fisher matrix are proportional to the magnitude of ionization power spectra — the smaller the magnitude, the smaller degradation factor and the more accurate is the cosmological parameter measurement. Figure 2 illustrates that the ionization power spectrum generically falls off at large k such that a relatively large core, which is more sensitive

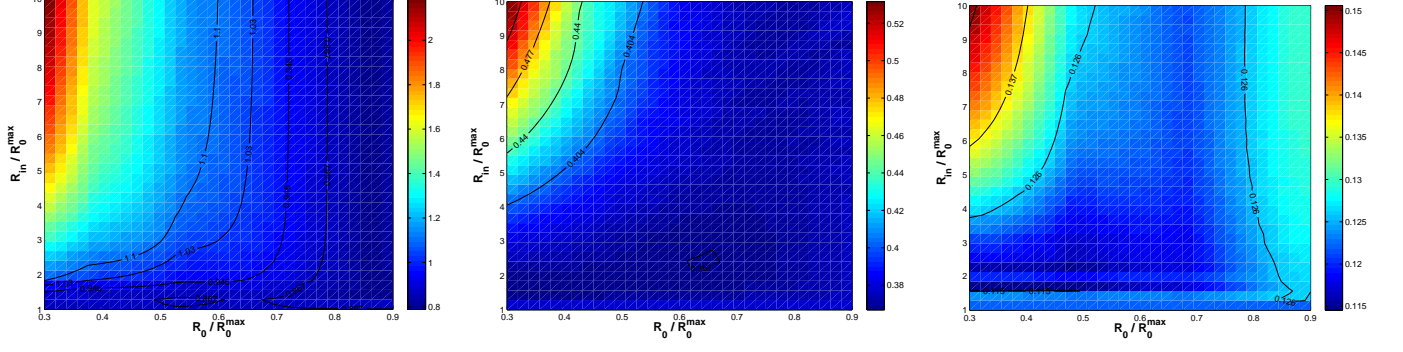


FIG. 7: 1σ error for m_ν marginalized over vanilla parameters for various configuration (R_0, R_{in}) of LOFAR(left panel), MWA(middle panel) and SKA(right panel). We made the same assumptions here as in Table V but assume only OPT model.

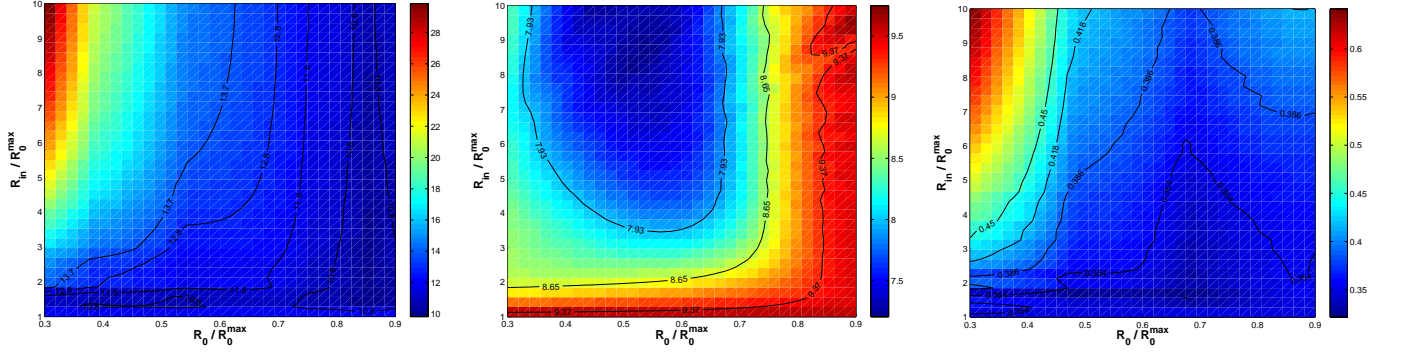


FIG. 8: Same as Figure 7 but for MID model. Figures are for LOFAR(left panel), MWA(middle panel) and SKA(right panel).

to these large k , may actually improve parameter constraints. This factor appears to be important for MWA because, as Figure 4 shows, a compactified MWA only occupies a rather narrow band in k -space. This means that MWA has to expand significantly in order to use much more large k modes.

It came to our attention that Lidz et al. [38] performed an analysis of the optimal configuration for MWA. Lidz et al. [38] concludes that the optimal layout for MWA is a giant core. This conclusion is slight different than ours; we find a compact but not exactly a giant core is optimal for MWA. The work in [38] defines the optimal configuration to be the configuration that maximizes the total signal-to-noise, while our definition is based on parameter constraints. In addition, the conclusion in [38] is based on the comparison of a giant core array configuration to one without a giant core, while we investigate a range of plausible configurations. It should be pointed out that both approaches should be tested with detailed simulations.

F. Varying collecting area

The survey volume and the noise per pixel are both affected by changing the collecting area A_e because the solid angle a survey observes is $\Omega \approx \lambda^2/A_e$ and $P^N \propto 1/A_e^2$ (Eq. (33)). For noise-dominated experiments, $\delta P_{\Delta T}/P_{\Delta T} \propto P^N/\sqrt{N_c} \propto A_e^{-2}/\sqrt{A_e^{-1}} = A_e^{-3/2}$, and, for signal-dominated experiments, $\delta P_{\Delta T}/P_{\Delta T} \propto 1/\sqrt{N_c} \propto A_e^{1/2}$. If we parametrize the scaling of the error on a cosmological parameter as $\Delta p \propto (A_e)^\beta$, we have $-1.5 < \beta < 0.5$. A caveat is FFTT which has fixed $\Omega = 2\pi$, so $\delta P_{\Delta T}/P_{\Delta T} \propto A_e^0$ (signal dominated) or $\delta P_{\Delta T}/P_{\Delta T} \propto 1/A_e^2$ (noise dominated). Since FFTT is nearly signal dominated, $\beta \lesssim 0$ for FFTT.

We show how collecting area affects the accuracy in Table X (OPT model) and XI (MID model). In the OPT model, it appears that $\beta \approx -0.4$ for LOFAR, $|\beta| \lesssim 0.2$ for MWA, $|\beta| \lesssim 0.3$ for SKA, and $\beta \sim -0.1$ for FFTT. In the MID model, it appears that $\beta \sim -1.3$ for LOFAR, $\beta \sim -0.5$ for MWA, $\beta \sim -0.6$ for SKA, $\beta \sim -0.3$ for FFTT. These exponents are compatible with the above arguments. The upshot is that varying A_e does not sig-

TABLE X: How cosmological constraints depend on collecting areas in the OPT model. Same assumptions as in Table V but for different collecting areas A_e and assume only OPT model. The exponent β tells the rule of thumb of the A_e -dependence of marginalized errors Δp , assuming $\Delta p \propto (A_e)^\beta$.

| | $A_e/A_e^{\text{fid}}{}^a$ | $\Delta\Omega_\Lambda$ | $\Delta \ln(\Omega_m h^2)$ | $\Delta \ln(\Omega_b h^2)$ | Δn_s | $\Delta \ln A_s$ |
|-------|----------------------------|------------------------|----------------------------|----------------------------|--------------|------------------|
| LOFAR | 2.0 | 0.020 | 0.24 | 0.40 | 0.048 | 0.80 |
| | 1 | 0.025 | 0.27 | 0.44 | 0.063 | 0.89 |
| | 0.5 | 0.039 | 0.40 | 0.62 | 0.10 | 1.3 |
| | β | -0.48 | -0.37 | -0.32 | -0.53 | -0.35 |
| MWA | 2.0 | 0.057 | 0.11 | 0.22 | 0.021 | 0.41 |
| | 1 | 0.046 | 0.11 | 0.19 | 0.022 | 0.37 |
| | 0.5 | 0.042 | 0.11 | 0.19 | 0.027 | 0.37 |
| | β | 0.22 | 0 | 0.11 | -0.18 | 0.07 |
| SKA | 2.0 | 0.0027 | 0.048 | 0.099 | 0.0077 | 0.19 |
| | 1 | 0.0038 | 0.044 | 0.083 | 0.0079 | 0.16 |
| | 0.5 | 0.0043 | 0.043 | 0.076 | 0.0089 | 0.15 |
| | β | -0.34 | 0.08 | 0.19 | -0.10 | 0.17 |
| FFTT | 2.0 | 0.00014 | 0.0031 | 0.0082 | 0.00037 | 0.015 |
| | 1 | 0.00015 | 0.0032 | 0.0084 | 0.00040 | 0.015 |
| | 0.5 | 0.00017 | 0.0035 | 0.0086 | 0.00046 | 0.016 |
| | β | -0.14 | -0.09 | -0.03 | -0.16 | -0.05 |

^a A_e^{fid} refers to the fiducial values assumed in Table IV and are not the same for different arrays.

TABLE XI: How cosmological constraints depend on collecting areas in the MID model. Same assumptions as in Table V but for different collecting areas A_e and assume only MID model. The exponent β tells the rule of thumb of the A_e -dependence of marginalized errors Δp , assuming $\Delta p \propto (A_e)^\beta$.

| | A_e/A_e^{fid} | $\Delta\Omega_\Lambda$ | $\Delta \ln(\Omega_m h^2)$ | $\Delta \ln(\Omega_b h^2)$ | Δn_s | $\Delta \ln A_s$ |
|-------|------------------------|------------------------|----------------------------|----------------------------|--------------|------------------|
| LOFAR | 2.0 | 0.086 | 0.044 | 0.072 | 0.15 | 0.35 |
| | 1 | 0.13 | 0.083 | 0.15 | 0.36 | 0.80 |
| | 0.5 | 0.26 | 0.17 | 0.35 | 0.92 | 2.0 |
| | β | -0.80 | -0.98 | -1.1 | -1.3 | -1.3 |
| MWA | 2.0 | 0.21 | 0.015 | 0.025 | 0.073 | 0.61 |
| | 1 | 0.22 | 0.017 | 0.029 | 0.097 | 0.76 |
| | 0.5 | 0.26 | 0.026 | 0.045 | 0.16 | 1.3 |
| | β | -0.15 | -0.40 | -0.42 | -0.57 | -0.55 |
| SKA | 2.0 | 0.013 | 0.0049 | 0.0079 | 0.0092 | 0.032 |
| | 1 | 0.014 | 0.0049 | 0.0081 | 0.012 | 0.037 |
| | 0.5 | 0.016 | 0.0063 | 0.011 | 0.022 | 0.053 |
| | β | -0.15 | -0.18 | -0.24 | -0.63 | -0.36 |
| FFTT | 2.0 | 0.00036 | 0.00037 | 0.00061 | 0.00032 | 0.0012 |
| | 1 | 0.00041 | 0.00038 | 0.00062 | 0.00036 | 0.0013 |
| | 0.5 | 0.00052 | 0.00041 | 0.00066 | 0.00046 | 0.0016 |
| | β | -0.27 | -0.07 | -0.06 | -0.26 | -0.21 |

nificantly affect parameter constraints.

G. Varying observation time and system temperature

The detector noise is affected by changing the observation time and system temperature. From Eq. (33), the noise $P^N \propto T_{\text{sys}}^2/t_0$. Therefore, for noise dominated experiments, $\delta P_{\Delta T}/P_{\Delta T} \propto P^N/\sqrt{N_c} \propto T_{\text{sys}}^2/t_0$, and for signal dominated experiments, $\delta P_{\Delta T}/P_{\Delta T} \propto 1/\sqrt{N_c} \propto (T_{\text{sys}}^2/t_0)^0$. Assuming that errors in cosmological param-

TABLE XII: How cosmological constraints depend on observation time in the OPT model. Same assumptions as in Table V but for different observation time t_0 and assume only OPT model. The exponent ϵ tells the rule of thumb of the t_0 -dependence of marginalized errors Δp , assuming $\Delta p \propto (t_0)^{-\epsilon}$. t_0 is in units of 4000 hours.

| | t_0 | $\Delta\Omega_\Lambda$ | $\Delta \ln(\Omega_m h^2)$ | $\Delta \ln(\Omega_b h^2)$ | Δn_s | $\Delta \ln A_s$ |
|-------|------------|------------------------|----------------------------|----------------------------|--------------|------------------|
| LOFAR | 4.0 | 0.014 | 0.17 | 0.28 | 0.034 | 0.56 |
| | 1 | 0.025 | 0.27 | 0.44 | 0.063 | 0.89 |
| | 0.25 | 0.055 | 0.56 | 0.88 | 0.14 | 1.8 |
| | ϵ | 0.49 | 0.43 | 0.41 | 0.51 | 0.42 |
| MWA | 4.0 | 0.040 | 0.081 | 0.16 | 0.015 | 0.29 |
| | 1 | 0.046 | 0.11 | 0.19 | 0.022 | 0.37 |
| | 0.25 | 0.059 | 0.15 | 0.27 | 0.038 | 0.52 |
| | ϵ | 0.14 | 0.22 | 0.19 | 0.34 | 0.21 |
| SKA | 4.0 | 0.0019 | 0.034 | 0.070 | 0.0054 | 0.13 |
| | 1 | 0.0038 | 0.044 | 0.083 | 0.0079 | 0.16 |
| | 0.25 | 0.0060 | 0.061 | 0.11 | 0.013 | 0.21 |
| | ϵ | 0.41 | 0.21 | 0.16 | 0.32 | 0.17 |
| FFTT | 4.0 | 0.00014 | 0.0031 | 0.0082 | 0.00037 | 0.015 |
| | 1 | 0.00015 | 0.0032 | 0.0084 | 0.00040 | 0.015 |
| | 0.25 | 0.00017 | 0.0035 | 0.0086 | 0.00046 | 0.016 |
| | ϵ | 0.07 | 0.04 | 0.02 | 0.08 | 0.02 |

TABLE XIII: How cosmological constraints depend on observation time in the MID model. Same assumptions as in Table V but for different observation time t_0 and assume only MID model. The exponent ϵ tells the rule of thumb of the t_0 -dependence of marginalized errors Δp , assuming $\Delta p \propto (t_0)^{-\epsilon}$. t_0 is in units of 4000 hours.

| | t_0 | $\Delta\Omega_\Lambda$ | $\Delta \ln(\Omega_m h^2)$ | $\Delta \ln(\Omega_b h^2)$ | Δn_s | $\Delta \ln A_s$ |
|-------|------------|------------------------|----------------------------|----------------------------|--------------|------------------|
| LOFAR | 4.0 | 0.061 | 0.031 | 0.051 | 0.11 | 0.25 |
| | 1 | 0.13 | 0.083 | 0.15 | 0.36 | 0.80 |
| | 0.25 | 0.36 | 0.24 | 0.50 | 1.3 | 2.9 |
| | ϵ | 0.64 | 0.74 | 0.82 | 0.89 | 0.88 |
| MWA | 4.0 | 0.15 | 0.010 | 0.017 | 0.052 | 0.43 |
| | 1 | 0.22 | 0.017 | 0.029 | 0.097 | 0.76 |
| | 0.25 | 0.36 | 0.037 | 0.064 | 0.23 | 1.8 |
| | ϵ | 0.32 | 0.47 | 0.48 | 0.54 | 0.52 |
| SKA | 4.0 | 0.0089 | 0.0035 | 0.0056 | 0.0065 | 0.022 |
| | 1 | 0.014 | 0.0049 | 0.0081 | 0.012 | 0.037 |
| | 0.25 | 0.023 | 0.0090 | 0.015 | 0.031 | 0.075 |
| | ϵ | 0.34 | 0.34 | 0.36 | 0.56 | 0.44 |
| FFTT | 4.0 | 0.00036 | 0.00037 | 0.00061 | 0.00032 | 0.0012 |
| | 1 | 0.00041 | 0.00038 | 0.00062 | 0.00036 | 0.0013 |
| | 0.25 | 0.00052 | 0.00041 | 0.00066 | 0.00046 | 0.0016 |
| | ϵ | 0.13 | 0.04 | 0.03 | 0.13 | 0.10 |

eter $\Delta p \propto (T_{\text{sys}}^2/t_0)^\epsilon$, we have $0 < \epsilon < 1$.

Since T_{sys}^2 and t_0^{-1} shares the same exponent, we evaluate the ϵ by varying only t_0 in Table XII (OPT model) and XIII (MID model). It appears that in average $\epsilon \sim 0.5$ for LOFAR, $\epsilon \sim 0.3$ for MWA, $\epsilon \sim 0.3$ for SKA, $\epsilon < 0.1$ for FFTT in the OPT model, and $\epsilon \sim 0.8$ for LOFAR, $\epsilon \sim 0.5$ for MWA, $\epsilon \sim 0.4$ for SKA, $\epsilon \lesssim 0.1$ for FFTT in the MID model. These exponents are compatible with the expected $0 < \epsilon < 1$ from the above argument. The upshot is that the order unity changes in T_{sys} and t_0 play a marginal role in the accuracy for future signal-dominated experiments.

H. Varying foreground cutoff scale k_{\min}

Finally, we test how accuracy is affected by varying k_{\min} above which foregrounds can be cleaned from the signal. One expects that the constraints tend to approach asymptotic values at small enough k_{\min} . However, the most effectively constrained modes are at small k ($k \sim 0.1 \text{ Mpc}^{-1}$) for noise dominated experiments, while the contributions from larger k modes are more important for cosmic variance limited experiments. This means that k_{\min} affects the noise dominated experiments most. Left panel of Figure 6 illustrates this by plotting cosmological constraints as a function of the relative minimum cutoff $\kappa_{\min} \equiv k_{\min} \times y(z)B(z)/2\pi$ which is a constant scale factor for all z -bins by definition. The slopes at $\kappa_{\min} = 1$ are rather large for MWA (varying from $\kappa_{\min} = 0.5$ to 2, the error in n_s varies from 0.032 to 0.39, about 10 times larger). For a signal dominated experiment like SKA, the constraints can be off by a factor of 3, or FFTT by a factor of 1.6. This suggests that in general k_{\min} is among top factors to affect cosmological constraints.

IV. CONCLUSION & OUTLOOK

A. Which assumptions matter most?

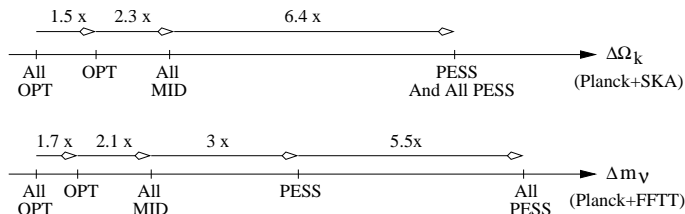


FIG. 9: Cartoon showing how cosmological parameter measurement accuracy depends on various assumptions. The cases labeled merely “PESS” or “OPT” have the PESS/OPT ionization power spectrum modeling with MID assumptions for everything else.

In Section III, we have quantified how cosmological parameter measurement accuracy depends on assumptions about ionization power modeling, reionization history, redshift range, experimental specifications such as the array configuration, and astrophysical foregrounds. We now return to the overarching question from Section I that motivated our study: among these assumptions, which make the most and least difference?

To quantify this, we consider two of the parameters for which 21cm tomography has the most potential for improving on Planck CMB constraints based on our estimates: Ω_k and m_ν . Figure 9 shows $\Delta\Omega_k$ based on data from Planck plus SKA as well as Δm_ν from Planck plus FFTT. Varying the ionization power modeling from PESS to OPT models improves the constraints on these two parameters by a factor of 6–15. From 21cm data

alone in the OPT model, the optimal array configuration can affect accuracies up to a factor 3 (Figure 7), redshift ranges affect it by up to a factor of 5 (Table VII), and residual foregrounds affect it by up to a factor of 10 (Figure 6, left panel). In summary, the assumptions can be crudely ordered by importance as ionization power modeling \gg foregrounds \sim redshift ranges \sim array layout $> A_e \sim T_{\text{sys}} \sim t_0 \sim k_{\text{max}} \sim$ non-Gaussianity.

B. Outlook

We have investigated how the measurement of cosmological parameters from 21 cm tomography depends on various assumptions. We have found that the assumptions about how well the reionization process can be modeled are the most important, followed by the assumptions pertaining to array layout, IGM evolution, and foreground removal.

Our results motivate further theoretical and experimental work. On the theoretical side, it will be valuable to develop improved EoR data analysis techniques. The OPT approach is restricted to when neutral fraction fluctuations are not important, which is not an accurate approximation during the EOR. On the other hand, although the PESS approach is in principle insensitive to our poor understanding of reionization by marginalizing over it, in practice this approach destroys too large a fraction of the cosmological information to be useful. Hopefully more detailed EoR simulations will enable our MID approach to be further improved into a phenomenological parametrization of our ignorance that is robust enough to be reliable, yet minimizes the loss of cosmological information.³

On the experimental side, there are numerous complications that are beyond the scope of this paper, but that are important enough to deserve detailed investigation in future work. To what extent can radio-frequency interference be mitigated, and to what extent does it degrade cosmological parameter accuracy? This is particularly important for instruments in densely populated parts of the world, such as LOFAR. To what extent is the subtraction of the foreground point sources hampered by the complicated off-center frequency scaling of the synthesized beam? To what extent does the dramatic variation of the synchrotron brightness temperature across the sky affect our results and optimal array design? Performing a realistic end-to-end simulation of possible experiments (from sky signal to volts and back) should be able to settle all of these issues.

These are difficult questions, but worthwhile because the potential for probing fundamental physics with 21 cm tomography is impressive: a future square kilometer

³ It is also possible to constrain cosmological parameters using lensing of 21cm fluctuations [46, 47, 48, 49].

array optimized for 21 cm tomography could improve the sensitivity of the Planck CMB satellite to spatial curvature and neutrino masses by up to two orders of magnitude, to $\Delta\Omega_k \approx 0.0002$ and $\Delta m_\nu \approx 0.007$ eV, and detect at 4σ the running of the spectral index predicted by the simplest inflation models.

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APPENDIX A: χ^2 GOODNESS OF FIT IN THE MID MODEL

In this appendix, we elucidate some issues in separating cosmological information from astrophysics in the MID model, and give the χ^2 goodness-of-fit test.

The parametrization of ionization power spectra is based on the assumption that these power spectra are smooth functions of k , and therefore can be parametrized with as many parameters as necessary to fit the data at some accuracy. However, the separation of cosmology from astrophysics implicitly depends on another assumption that the shapes of ionization power spectra are distinguishable from that of matter power spectrum, since one can only measure the *total* 21cm power spectrum. Albeit sometimes the shape may be similar at small k (see the plateaus in the ratios of power spectra in Figure 2), the slope and amplitude of ionization power spectrum at the fall-off region can in principle distinguish nuisance functions from the matter power spectrum, determine the overall amplitude, and in return use the data at small k to further constrain the nuisance parameters that correspond to the amplitudes.

An avalanche of data from upcoming 21cm experiments can make it possible to justify the MID model with

some parametrization of ionization power spectra. There are standard statistical methods for testing whether the parametrization is successful. We now give a compact description of the χ^2 goodness-of-fit test, and refer interested readers to [41] for a useful review on the statistics. Note that we did not implement the χ^2 test in this paper since this would need observational data. The description of χ^2 test below is intended to complement the discussions of the MID model in the main part of this paper. We want to test the hypothesis H_0 that the parametrization with fitting parameter values is an accurate account of the ionization power spectra. The parameter vector to be fitted is $\Theta \equiv (\lambda_i (i = 1, \dots, N_p), \beta_\alpha (\alpha = 1, \dots, n_{\text{ion}}))$, where N_p and n_{ion} are the number of cosmological and ionization parameters, respectively. The observed data vector is $\mathbf{y} \equiv (y_1, \dots, y_N)$ where $y_i \equiv P_{\Delta T}(\mathbf{k}_i)$ at each pixel \mathbf{k}_i labeled by $i = 1, \dots, N$, where N is the total number of pixels. Assuming the Gaussian statistic in the measurements, the corresponding vector \mathbf{F} for the expected value is $F(\mathbf{k}_i; \Theta) = (\mathcal{P}_{\delta\delta} - 2\mathcal{P}_{x\delta} + \mathcal{P}_{xx}) + 2(\mathcal{P}_{\delta\delta} - \mathcal{P}_{x\delta})\mu^2 + \mathcal{P}_{\delta\delta}\mu^4$, and the variance is $\sigma_i^2 \equiv (\delta P_{\Delta T}(\mathbf{k}_i))^2 = \frac{1}{N_c} [P_{\Delta T}(\mathbf{k}_i) + P_N(k_{i\perp})]^2$. We can now compute χ^2 :

$$\chi^2(\Theta) = (\mathbf{y} - \mathbf{F}(\Theta))^T C^{-1} (\mathbf{y} - \mathbf{F}(\Theta)), \quad (\text{A1})$$

where C is the covariance matrix. If each measurement y_i is independent, then C becomes diagonal with $C_{ii} = \sigma_i^2$. Then Eq. (A1) is simplified to be

$$\chi^2(\Theta) = \sum_{i=1}^N \frac{[y_i - F(\mathbf{k}_i; \Theta)]^2}{\sigma_i^2}. \quad (\text{A2})$$

We can define the p -value as the probability, under the assumption of the hypothesis H_0 , of obtaining data at least as incompatible with H_0 as the data actually observed. So

$$p = \int_{\chi^2(\Theta)}^{\infty} f(z; n_d) dz, \quad (\text{A3})$$

where $f(z; n_d)$ is the χ^2 probability density function (p.d.f.) with n_d degrees of freedom $n_d = N - (N_p + n_{\text{ion}})$. Values of the χ^2 p.d.f. can be obtained from the CERN-LIB routine PROB [42]. To set the criterion, a fit is good if $p \geq 0.95$, i.e. the real data fit the parametrization better than the 95% confidence level.

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